

Mathematics scope and sequence chart: Grades 10 to 12 advanced: Quantitative methods

	Grade 10	Grade 11	Grade 12
REASONING AND PROBLEM SOLVING			
To be applied to all strands	<ul style="list-style-type: none"> Routine and non-routine problem solving Modelling real-world applications Identifying and using connections between mathematical topics Breaking complex problems into smaller tasks; using problem solving strategies to set up and solve relevant equations and to perform appropriate calculations Developing and explaining short chains of reasoning, using correct notation and terms; generalising; generating mathematical proofs; identifying exceptional cases Solving problems systematically; conjecturing possibilities; synthesising, presenting, interpreting and criticising mathematical information; working to expected degrees of accuracy Recognising when to use ICT; using ICT efficiently 	<ul style="list-style-type: none"> Routine and non-routine problem solving Modelling real-world applications Identifying and using connections between mathematical topics Breaking complex problems into smaller tasks; using problem solving strategies to set up and solve relevant equations and to perform appropriate calculations Developing and explaining chains of reasoning, using correct notation and terms; generalising; generating mathematical proofs; discussing exceptional cases Solving problems systematically; conjecturing possibilities; synthesising, presenting, interpreting and criticising mathematical information; working to expected degrees of accuracy Recognising when to use ICT; using ICT efficiently 	<ul style="list-style-type: none"> Routine and non-routine problem solving Modelling statistical situations Identifying and using connections between mathematical topics Breaking complex problems into smaller tasks; using problem solving strategies to set up and solve relevant equations and to perform appropriate calculations Developing and explaining chains of reasoning, using correct notation and terms confidently; generalising; discussing exceptional cases and statistical outliers Solving problems systematically; conjecturing possibilities; synthesising, presenting, interpreting and criticising mathematical information; working to expected degrees of accuracy Recognising when to use ICT; using ICT efficiently
NUMBER AND ALGEBRA			
General	<ul style="list-style-type: none"> Real-world numerical and algebraic applications Linking algebraic reasoning to geometrical ideas Contributions to mathematics by Islamic scholars 	<ul style="list-style-type: none"> Real-world numerical and algebraic applications 	<ul style="list-style-type: none"> Real-world numerical and algebraic applications
Number	<ul style="list-style-type: none"> Powers, nth roots; exact calculations with surds; standard form Calculations with any real numbers, including mental calculations; multiplicative nature of proportional reasoning; using, forming, simplifying and comparing ratios; percentage calculations, including percentage of a percentage, inverse percentage; compound interest Evaluating recurring decimal as an exact fraction (see also Geometric sequences) 	<ul style="list-style-type: none"> Limiting value of compounding interest more and more frequently 	<ul style="list-style-type: none"> Laws of indices and of logarithms in any base The number e and the series expansion for e^x Using appropriate function keys on a scientific calculator to work with indices, logarithms and exponentials

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Set theory	<ul style="list-style-type: none"> The number sets: \mathbb{R} (reals), \mathbb{Z} (integers), \mathbb{N} (natural numbers), \mathbb{Q} (rationals); irrational numbers Common set theory symbols: \mathcal{E} (universal set), \emptyset (null set), \in (is a member of), \notin (is not a member of), \forall (for all), brace notation; $A \cup B$ (union of sets); $A \cap B$ (intersection of sets); A' (complement of set A); $A \cup A' = \mathcal{E}$; further Venn diagrams 	<ul style="list-style-type: none"> Solution sets of equations and inequalities 	
Sequences, functions and graphs	<ul style="list-style-type: none"> Algebraic generalisations from odd and even numbers Sequences from term-to-term and position-to-term definitions; simple growth patterns; Pascal's triangle; arithmetic sequences; sum of first n consecutive positive integers Geometric sequences and their sums; recurring decimal as an example of infinite geometric series Function, domain and range; functional relationships between related variables; graphs of simple functional relationships from familiar contexts; recognising when a graph represents a function; function notation $y = f(x)$ Translating 'y is proportional to x' into equation $y = kx$, representing a straight line through the origin with gradient k; common examples of direct proportion; quadratic proportion Plotting straight line equations $y = mx + c$; m as gradient of line and c as intercept on y-axis; establishing Cartesian equations of lines from appropriate information; conditions for two straight lines to be parallel or perpendicular, including special cases; implicit form $ax + by + d = 0$ Finding point of intersection of two lines: exactly using algebraic methods, approximately using graphical methods; interpreting solutions in physical contexts Tangent line at a point on the graph of a function, its gradient and its interpretation in physical applications Regions of linear inequality; simple quadratic inequalities Quadratic functions of the form $y = ax^2 + bx + c$; their graphs, intercepts with the coordinate axes, axis of symmetry and coordinates of the maximum or minimum point; modelling with quadratic functions 	<ul style="list-style-type: none"> Finite and infinite convergent geometric sequences Sums of first n squares and cubes; further work on sequences, series; recurrence relations; arrangements; sigma notation Binomial theorem and binomial coefficients Combinations and permutations Odd and even functions; symmetry properties Further quadratic functions $y = ax^2 + bx + c$; their graphs, intercepts with coordinate axes, axis of symmetry and coordinates of maximum or minimum point; when such functions are increasing, decreasing or stationary; modelling situations with quadratic functions Approximate solutions of $ax^2 + bx + c = 0$ from graph Translating 'y is inversely proportional to x' into equation $y = k/x$, where $x \neq 0$ and x- and y-axes are each asymptotes to the curve; examples of inverse proportion Using physical contexts to plot and interpret graphs of linear, quadratic, cubic, reciprocal, sine and cosine functions, modulus function and simple non-standard functions; using a graphics calculator to find approximate solutions to physical problems Inverse functions of simple functions Composite functions; notation $y = f(g(x))$; deconstruction of composite functions into constituent functions Transformation of $y = f(x)$ to $y = f(x) + a$, $y = f(x + a)$, $y = af(x)$, $y = f(ax)$, and interpretations as translation, stretch or compression Exponential growth and decay and associated graphs $y = a^x$, where $a > 0$; using graphics calculator to plot graphs of exponential function, e^x, and natural logarithm function, $\ln x$; solution of equation $y = a^x$ and its use in problems 	<ul style="list-style-type: none"> Further algebraic manipulation, factorisation and simplification Partial fractions The remainder theorem and factor theorem Key features of functions: <ul style="list-style-type: none"> polynomial functions rational functions exponential and logarithm functions circular functions modulus function Symmetry of functions Inverse functions Transforming and combining functions

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Expressions, equations and formulae	<ul style="list-style-type: none"> Working with symbols; distinguishing expressions, equations, formulae and identities; recognising that rules of algebra generalise the rules of arithmetic Brackets and correct order of precedence of operations when performing numerical or algebraic calculations Combining numeric or algebraic fractions Multiplication of combinations of monomial, binomial and trinomial expressions, including squares of linear binomial expressions; linear factors of quadratic expressions; factorisation of difference of two squares Simplifying numeric and algebraic fractions; rationalising denominators of fractions containing surds Solving any linear equation with one unknown, and a pair of simultaneous linear equations Exact solutions of quadratic equations by factorisation, by completing the square, by using the quadratic formula Rearranging formulae connecting at least two variables Substituting values in formulae and expressions; substituting an expression into another formula 	<ul style="list-style-type: none"> Solving equations and inequalities associated with all of the above functions The quadratic equation revisited: use of discriminant; number of real roots Solution set of two simultaneous equations, one linear and one quadratic; physical problems modelled simultaneously by two such functions 	
Limits		<ul style="list-style-type: none"> Exploring limits in various contexts Defining the tangent and its slope at a point on a curve 	
Differential calculus		<ul style="list-style-type: none"> First principles definition of a derivative and its interpretation as a rate of change Derivative of the standard functions given above 	<ul style="list-style-type: none"> Second order derivatives Derivative of the standard functions given above Use of the derivative to analyse behaviour of functions Derivatives of combinations of functions Use of the derivative to maximise or minimise a function
Integral calculus			<ul style="list-style-type: none"> Integration as the 'inverse' of differentiation The indefinite integral The definite integral; its interpretation as an area function and as a measure or probability under a distribution curve Use of trapezium rule as approximation to definite integral Integrals of x^n (including $n = -1$) and e^{kx} Some techniques of integration
Using ICT	<ul style="list-style-type: none"> Using ICT to explore number and algebra 	<ul style="list-style-type: none"> Using ICT to explore number and algebra 	<ul style="list-style-type: none"> Using ICT to explore number and algebra

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GEOMETRY AND MEASURES			
General	<ul style="list-style-type: none"> Real-world geometrical applications, including Islamic patterns 	<ul style="list-style-type: none"> Real-world geometrical applications 	
Using ICT	<ul style="list-style-type: none"> Using dynamic geometry systems (DGS) to explore pattern, similarity, congruence and constructions, and to conjecture geometric properties and theorems 	<ul style="list-style-type: none"> Using dynamic geometry systems (DGS) to explore pattern, similarity, congruence and constructions, and to conjecture geometric properties and theorems 	
Geometry	<ul style="list-style-type: none"> Angles at a point, angles on a straight line, alternate and corresponding angles; formal arguments to establish congruency of two triangles; using congruency of two triangles to generate further knowledge Similarity of two triangles and other rectilinear shapes; preservation of shape and angles, but not of size; ratio of lengths of sides and areas of similar figures; ratio of volume of a scale model to volume of the actual object Regular polygons and their interior and exterior angles Proof that the perpendicular from the centre of a circle to a chord bisects the chord, and that two tangents from an external point to a circle are of equal length Constructions using straight edge and compass Simple loci, including those arising in physical situations 	<ul style="list-style-type: none"> Points of intersection of straight line with circle Relevant vocabulary associated with a circle; proof of standard circle theorems Transformations of rectilinear figures using combinations of translations, rotations about centre of rotation, enlargements about centre of enlargement, and reflections about a line; positive, negative and fractional scale factors in enlargements Plans and elevations 	
Trigonometry	<ul style="list-style-type: none"> Solution of triangles using standard trigonometric ratios Proof of Pythagoras' theorem; using Pythagoras' theorem to find Pythagorean triples, distance between two points, set up the Cartesian equation of a circle of radius r, centred at point (α, β); unit circle $x^2 + y^2 = 1$ and graphs of circular functions $\sin \theta$ and $\cos \theta$ for any angle θ°, where $0^\circ \leq \theta^\circ \leq 360^\circ$; simple problems modelled by circular functions 	<ul style="list-style-type: none"> Sine rule and cosine rule; triangle problems in two and three dimensions; area of triangle using $\frac{1}{2} ab \sin C$ Using Pythagoras' theorem to show that $\sin^2 \theta^\circ + \cos^2 \theta^\circ \equiv 1$ for any angle θ°; simple related identities; problems modelled by circular functions Plots of graphs of circular functions $\sin \theta$ and $\cos \theta$ for any angle θ, using radian measure (see also Sequences, functions and graphs) 	

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Measures and mensuration	<ul style="list-style-type: none"> • SI units • Perimeters and areas of rectilinear and circular shapes, and volumes of rectilinear solids, cones, cylinders and spheres • Radian measure; sector areas and arc lengths • Bearings; latitude, longitude and great circles and their use in solving problems relating to position, distance and displacement on the Earth's surface • Compound measures, including those that reinforce links with science and technology 	<ul style="list-style-type: none"> • Further work on rates and other compound measures, including those that reinforce links with other disciplines, science, technology and the social sciences 	
Vectors		<ul style="list-style-type: none"> • Vectors: position vector and translation as a vector displacement; knowing that the vector displacement depends only on the starting point and the finish point, and not on intermediate steps • Addition and subtraction of two vectors in up to three dimensions and the corresponding vector diagrams • Scalar product of two vectors; multiplication of a vector by a scalar; magnitude and direction of a vector; vector displacement and velocity; unit vectors and components • Solution of physical problems using vectors 	

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PROBABILITY AND STATISTICS			
Probability and statistics	<ul style="list-style-type: none"> Using statistical data collected from samples to make inferences about the population as a whole Representative samples; random and biased samples; location of sources of bias Random variables Distinguishing qualitative from quantitative data, and discrete from continuous data Planning surveys and questionnaires to collect meaningful primary data from samples to make estimates of, or test hypotheses about, quantities or attributes characteristic of the population as a whole Using secondary data from published sources, including the Internet Measures of central tendency Measures of spread Histograms, frequency and (relative) frequency distributions and associated distributions; using grouped continuous data Stem-and-leaf diagrams and box-and-whisker plots; making inferences and drawing conclusions from analysis of data in a range of situations Scatter diagrams between two random variables associated with common contexts; elementary qualitative discussion of correlation, including positive and negative correlation; drawing a line of best fit through the scatter points when there appears to be some correlation 	<ul style="list-style-type: none"> Empirical probability (relative frequency) of a particular value; using simple mathematical models to calculate theoretical probability of a particular outcome for a random variable; knowing that probability values lie between 0 and 1 Risk as probability of occurrence of an adverse event; risk in everyday situations Sum of probabilities for all outcomes of mutually exclusive and exhaustive events is 1; when two events A and B are mutually exclusive, probability of A or B, $P(A \cup B)$, is $P(A) + P(B)$; two events A and B are independent if the probability of A and B occurring together, $P(A \cap B)$, is $P(A) \times P(B)$; $P(A \cap B)$ is $P(A) \times P(B A)$ when B is conditional on A Tree diagrams for representing and calculating the probabilities of compound events when events are independent or when one is conditional on another Trends over time and moving averages Simulation using random numbers to model simple situations, including waiting times 	<ul style="list-style-type: none"> Classifying data Statistical measurement and measurement scales Sampling from a population and associated vocabulary Random numbers and their uses in sampling, measurement and calculating probabilities Random numbers and their uses in simulations Random variables Collecting and organising data: surveys and questionnaires statistical charts and distributions measures of central tendency and of spread linear coding for mean and variance percentiles, quartiles and range Laws of probability for single and combined events; sample space; tree diagrams Expectation of a random variable Properties of probability distributions Discrete probability distributions, including the binomial and the Poisson distributions; the Poisson distribution as an approximation to the binomial distribution The normal distribution and its properties The normal distribution as an approximation to the binomial distribution and to the Poisson distribution Linear correlation; correlation coefficient; least squares regression; rank correlation coefficient Making inferences from data: level of significance confidence interval null and alternative hypotheses chi-squared test
Using ICT	<ul style="list-style-type: none"> Using a calculator with statistical functions for analysing large data sets Using ICT packages to produce statistical tables and graphs 	<ul style="list-style-type: none"> Using a calculator with statistical functions for analysing large data sets Using ICT packages to produce statistical tables and graphs 	<ul style="list-style-type: none"> Using a calculator with statistical functions for analysing large data sets Using ICT packages to produce statistical tables and graphs