Mathematics scope and sequence chart: Grades 10 to 12 advanced: Mathematics for science

	Grade 10	Grade 11	Grade 12
REASONING AND PROBLEM SOLVING			
To be applied to all strands	• Routine and non-routine problem solving • Modelling real-world applications • Identifying and using connections between mathematical topics • Breaking complex problems into smaller tasks; using problem solving strategies to set up and solve relevant equations and to perform appropriate calculations • Developing and explaining short chains of reasoning, using correct notation and terms; generalising; generating mathematical proofs; identifying exceptional cases • Solving problems systematically; conjecturing possibilities; synthesising, presenting, interpreting and criticising mathematical information; working to expected degrees of accuracy • Recognising when to use ICT; using ICT efficiently	• Routine and non-routine problem solving • Modelling real-world applications • Identifying and using connections between mathematical topics • Breaking complex problems into smaller tasks; using problem solving strategies to set up and solve relevant equations and to perform appropriate calculations • Developing and explaining chains of reasoning, using correct notation and terms; generalising; generating mathematical proofs; discussing exceptional cases • Solving problems systematically; conjecturing possibilities; synthesising, presenting, interpreting and criticising mathematical information; working to expected degrees of accuracy • Recognising when to use ICT; using ICT efficiently	• Routine and non-routine problem solving • Modelling real-world applications; using error bounds • Identifying and using connections between mathematical topics • Breaking complex problems into smaller tasks; using problem solving strategies to set up and solve relevant equations and to perform appropriate calculations • Developing and explaining longer chains of reasoning, using logical implication and using correct notation and terms confidently; generalising; generating mathematical proofs and disproving by counter-example; discussing exceptional cases • Solving problems systematically; conjecturing possibilities; synthesising, presenting, interpreting and criticising mathematical information; working to expected degrees of accuracy • Recognising when to use ICT; using ICT efficiently
NUMBER AND ALGEBRA			
General	• Real-world numerical and algebraic applications • Linking algebraic reasoning to geometrical ideas • Contributions to mathematics by Islamic scholars	• Real-world numerical and algebraic applications	• Real-world numerical and algebraic applications
Number	• Powers, *n*th roots; exact calculations with surds; standard form • Calculations with any real numbers, including mental calculations; multiplicative nature of proportional reasoning; using, forming, simplifying and comparing ratios; percentage calculations, including percentage of a percentage, inverse percentage; compound interest • Evaluating recurring decimal as an exact fraction (see also Geometric sequences)	• Limiting value of compounding interest more and more frequently	• Laws of indices and of logarithms in any base • The number e • Using appropriate function keys on a scientific calculator to work with indices, logarithms and exponentials • Introduction to complex numbers and Argand diagram

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Set theory	• The number sets: $\mathbb{R}$ (reals), $\mathbb{Z}$ (integers), $\mathbb{N}$ (natural numbers), $\mathbb{Q}$ (rationals); irrational numbers • Common set theory symbols: $\mathcal{E}$ (universal set), $\varnothing$ (null set), $\in$ (is a member of), $\notin$ (is not a member of), $\forall$ (for all), brace notation; $A \cup B$ (union of sets); $A \cap B$ (intersection of sets); A' (complement of set A); $A \cup A' = \mathcal{E}$; further Venn diagrams	• Solution sets of equations and inequalities	
Sequences, functions and graphs	• Algebraic generalisations from odd and even numbers • Sequences from term-to-term and position-to-term definitions; simple growth patterns; Pascal's triangle; arithmetic sequences; sum of first n consecutive positive integers • Geometric sequences and their sums; recurring decimal as an example of infinite geometric series • Function, domain and range; functional relationships between related variables; graphs of simple functional relationships from familiar contexts; recognising when a graph represents a function; function notation $y = f(x)$ • Translating 'y is proportional to x' into equation $y = kx$, representing a straight line through the origin with gradient k; common examples of direct proportion; quadratic proportion • Plotting straight line equations $y = mx + c$; m as gradient of line and c as intercept on y-axis; establishing Cartesian equations of lines from appropriate information; conditions for two straight lines to be parallel or perpendicular, including special cases; implicit form $ax + by + d = 0$ • Finding point of intersection of two lines: exactly using algebraic methods, approximately using graphical methods; interpreting solutions in physical contexts • Tangent line at a point on the graph of a function, its gradient and its interpretation in physical applications • Regions of linear inequality; simple quadratic inequalities • Quadratic functions of the form $y = ax^2 + bx + c$; their graphs, intercepts with the coordinate axes, axis of symmetry and coordinates of the maximum or minimum point; modelling with quadratic functions	• Finite and infinite convergent geometric sequences • Sums of first n squares and cubes; further work on sequences, series; recurrence relations; arrangements; sigma notation • Binomial theorem and binomial coefficients • Combinations and permutations • Odd and even functions; symmetry properties • Further quadratic functions $y = ax^2 + bx + c$; their graphs, intercepts with coordinate axes, axis of symmetry and coordinates of maximum or minimum point; when such functions are increasing, decreasing or stationary; modelling situations with quadratic functions • Approximate solutions of $ax^2 + bx + c = 0$ from graph • Translating 'y is inversely proportional to x' into equation $y = k/x$, where $x \neq 0$ and x- and y-axes are each asymptotes to the curve; examples of inverse proportion • Using physical contexts to plot and interpret graphs of linear, quadratic, cubic, reciprocal, sine and cosine functions, modulus function and simple non-standard functions; using a graphics calculator to find approximate solutions to physical problems • Inverse functions of simple functions • Composite functions; notation $y = f(g(x))$; deconstruction of composite functions into constituent functions • Transformation of $y = f(x)$ to $y = f(x) + a$, $y = f(x + a)$, $y = af(x)$, $y = f(ax)$, and interpretations as translation, stretch or compression • Exponential growth and decay and associated graphs $y = a^x$, where $a > 0$; using graphics calculator to plot graphs of exponential function, e^x, and natural logarithm function, $\ln x$; solution of equation $y = a^x$ and its use in problems	• Further algebraic manipulation, factorisation and simplification • Partial fractions • The remainder theorem and factor theorem • Binomial series • Combinations and permutations • Key features of functions: polynomial functions rational functions exponential and logarithm functions circular functions modulus function • Composite functions; inverse functions • Symmetry of functions • Further transformations of functions • Periodic functions • Simple examples of curves given by parametric equations

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Expressions, equations and formulae	• Working with symbols; distinguishing expressions, equations, formulae and identities; recognising that rules of algebra generalise the rules of arithmetic • Brackets and correct order of precedence of operations when performing numerical or algebraic calculations • Combining numeric or algebraic fractions • Multiplication of combinations of monomial, binomial and trinomial expressions, including squares of linear binomial expressions; linear factors of quadratic expressions; factorisation of difference of two squares • Simplifying numeric and algebraic fractions; rationalising denominators of fractions containing surds • Solving any linear equation with one unknown, and a pair of simultaneous linear equations • Exact solutions of quadratic equations by factorisation, by completing the square, by using the quadratic formula • Rearranging formulae connecting at least two variables • Substituting values in formulae and expressions; substituting an expression into another formula	• Solving equations and inequalities associated with all of the above functions • The quadratic equation revisited: use of discriminant; number of real roots • Solution set of two simultaneous equations, one linear and one quadratic; physical problems modelled simultaneously by two such functions	
Limits		• Exploring limits in various contexts • Defining the tangent and its slope at a point on a curve	
Differential calculus		• First principles definition of a derivative and its interpretation as a rate of change • Derivative of the standard functions given above	• Higher order derivatives • Derivative of the standard functions given above • Use of the derivative to analyse behaviour of functions • Derivatives of combinations of functions and of composite functions • Use of the derivative in optimisation, in mechanics and in other physical examples, including geometry • Use of the derivative in numerical approximations and in approximations to functions and locating roots of equations

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Integral calculus			• Integration as the 'inverse' of differentiation • The indefinite integral • The definite integral; its interpretation as an area function • Use of trapezium rule as an approximation to a definite integral • Limits of integration and discussion of simple convergent integrals as an integration limit tends to infinity • Integrals of the standard functions • Some techniques of integration • Use of integrals in mechanics and in the solution of a range of physical problems, including geometry • Solution of simple differential equations and their use in mathematical modelling of situations
Using ICT	• Using ICT to explore number and algebra	• Using ICT to explore number and algebra	• Using ICT to explore number and algebra
GEOMETRY AND MEASURES			
General	• Real-world geometrical applications, including Islamic patterns	• Real-world geometrical applications	• Real-world geometrical applications
Using ICT	• Using dynamic geometry systems (DGS) to explore pattern, similarity, congruence and constructions, and to conjecture geometric properties and theorems	• Using dynamic geometry systems (DGS) to explore pattern, similarity, congruence and constructions, and to conjecture geometric properties and theorems	• Using a dynamic geometry system (DGS) for further exploration of geometry and motion
Geometry	• Angles at a point, angles on a straight line, alternate and corresponding angles; formal arguments to establish congruency of two triangles; using congruency of two triangles to generate further knowledge • Similarity of two triangles and other rectilinear shapes; preservation of shape and angles, but not of size; ratio of lengths of sides and areas of similar figures; ratio of volume of a scale model to volume of the actual object • Regular polygons and their interior and exterior angles • Proof that the perpendicular from the centre of a circle to a chord bisects the chord, and that two tangents from an external point to a circle are of equal length • Constructions using straight edge and compass • Simple loci, including those arising in physical situations	• Points of intersection of straight line with circle • Relevant vocabulary associated with a circle; proof of standard circle theorems • Transformations of rectilinear figures using combinations of translations, rotations about centre of rotation, enlargements about centre of enlargement, and reflections about a line; positive, negative and fractional scale factors in enlargements • Plans and elevations	

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Trigonometry	• Solution of triangles using standard trigonometric ratios • Proof of Pythagoras' theorem; using Pythagoras' theorem to find Pythagorean triples, distance between two points, set up the Cartesian equation of a circle of radius r, centred at point (α, β); unit circle $x^2 + y^2 = 1$ and graphs of circular functions $\sin\theta$ and $\cos\theta$ for any angle $\theta°$, where $0° \leq \theta° \leq 360°$; simple problems modelled by circular functions	• Sine rule and cosine rule; triangle problems in two and three dimensions; area of triangle using $\frac{1}{2}ab\sin C$ • Using Pythagoras' theorem to show that $\sin^2\theta° + \cos^2\theta° \equiv 1$ for any angle $\theta°$; simple related identities; problems modelled by circular functions • Plots of graphs of circular functions $\sin\theta$ and $\cos\theta$ for any angle θ, using radian measure (see also Sequences, functions and graphs)	• Trigonometric identities and equations
Measures and mensuration	• SI units • Perimeters and areas of rectilinear and circular shapes, and volumes of rectilinear solids, cones, cylinders and spheres • Radian measure; sector areas and arc lengths • Bearings; latitude, longitude and great circles and their use in solving problems relating to position, distance and displacement on the Earth's surface • Compound measures, including those that reinforce links with science and technology	• Further work on rates and other compound measures, including those that reinforce links with other disciplines, science, technology and the social sciences	• Further compound measures, especially those generated as a derivative or integral of some function used in context • Areas and volumes by integration
Vectors		• Vectors: position vector and translation as a vector displacement; knowing that the vector displacement depends only on the starting point and the finish point, and not on intermediate steps • Addition and subtraction of two vectors in up to three dimensions and the corresponding vector diagrams • Scalar product of two vectors; multiplication of a vector by a scalar; magnitude and direction of a vector; vector displacement and velocity; unit vectors and components • Solution of physical problems using vectors	• Vectors: velocity and acceleration vectors and their use in analysing motion • Speed as the magnitude of velocity; unit vectors and components • Scalar product of two vectors and its use to determine magnitudes and the angle between two vectors • Vector equation of a straight line • Solution of physical problems using vectors

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PROBABILITY AND STATISTICS			
Probability and statistics	• Using statistical data collected from samples to make inferences about the population as a whole • Representative samples; random and biased samples; location of sources of bias • Random variables • Distinguishing qualitative from quantitative data, and discrete from continuous data • Planning surveys and questionnaires to collect meaningful primary data from samples to make estimates of, or test hypotheses about, quantities or attributes characteristic of the population as a whole • Using secondary data from published sources, including the Internet • Measures of central tendency • Measures of spread • Histograms, frequency and (relative) frequency distributions and associated distributions; using grouped continuous data • Stem-and-leaf diagrams and box-and-whisker plots; making inferences and drawing conclusions from analysis of data in a range of situations • Scatter diagrams between two random variables associated with common contexts; elementary qualitative discussion of correlation, including positive and negative correlation; drawing a line of best fit through the scatter points when there appears to be some correlation	• Empirical probability (relative frequency) of a particular value; using simple mathematical models to calculate theoretical probability of a particular outcome for a random variable; knowing that probability values lie between 0 and 1 • Risk as probability of occurrence of an adverse event; risk in everyday situations • Sum of probabilities for all outcomes of mutually exclusive and exhaustive events is 1; when two events A and B are mutually exclusive, probability of A or B, $P(A \cup B)$, is $P(A) + P(B)$; two events A and B are independent if the probability of A and B occurring together, $P(A \cap B)$, is $P(A) \times P(B)$; $P(A \cap B)$ is $P(A) \times P(B \mid A)$ when B is conditional on A • Tree diagrams for representing and calculating the probabilities of compound events when events are independent or when one is conditional on another • Trends over time and moving averages • Simulation using random numbers to model simple situations, including waiting times	
Using ICT	• Using a calculator with statistical functions for analysing large data sets • Using ICT packages to produce statistical tables and graphs	• Using a calculator with statistical functions for analysing large data sets • Using ICT packages to produce statistical tables and graphs	