

### Summary of students' performance by the end of Grade 10

#### Reasoning and problem solving

Students solve routine and non-routine problems in a range of mathematical and other contexts. They use mathematics to model and predict outcomes of real-world applications. They break down complex problems into smaller tasks, and set up and perform appropriate calculations and manipulations. They identify and use connections between mathematical topics. They develop and explain short chains of logical reasoning, using correct mathematical notation and terms. They generate mathematical proofs and generalise when possible. They approach problems systematically, knowing when to enumerate all outcomes. They conjecture alternative possibilities with 'What if ...?' and 'What if not ...?' questions. They synthesise, present, interpret and criticise mathematical information, working to expected degrees of accuracy. They recognise when to use ICT and do so efficiently.

#### Number and algebra

Students identify and use number sets and set notation. They calculate with any real numbers, including powers, roots, and numbers expressed in standard form. They use proportional reasoning to solve problems involving scale, ratios and percentages, including compound interest. They are aware of the role of symbols in algebra. They generate and manipulate algebraic expressions, including algebraic fractions, equations and formulae. They multiply any two polynomial expressions and factorise quadratic expressions. They sum arithmetic and geometric sequences and investigate the growth of patterns, generalising relationships to model the behaviour of the pattern. They convert recurring decimals to exact fractions. They use algebraic methods to solve linear and quadratic equations, and a pair of simultaneous linear equations. They plot and interpret straight line and quadratic graphs, and graph regions of linear inequality. They use function notation and find the tangent at a point on the graph of a function. Through their study of linear and quadratic functions and their graphs, and the solution of the related equations, students begin to appreciate numerical and algebraic applications in the real world. They use realistic data and ICT to analyse problems.

#### Geometry and measures

Students use their knowledge of geometry, Pythagoras' theorem and the trigonometry of right-angled triangles to solve practical and theoretical problems relating to shape and space. They understand congruence and similarity. They prove that the perpendicular from the centre of a circle to a chord bisects the chord and that the two tangents from an external point to a circle are of equal length. They carry out straight edge and compass constructions and determine the locus of an object moving according to a rule. They use radians as a measure of angle, and dimensionally correct units for length, area and volume. They solve problems involving rates and compound measures. They use formulae to calculate the length of an arc and the area of a sector of a circle, the area of any triangle, trapezium, parallelogram or quadrilateral with perpendicular diagonals, and the surface area and volume of a right prism, cylinder, cone, sphere and pyramid. They

use bearings, latitude, longitude and great circles to solve problems relating to position, distance and displacement on the Earth's surface. They use ICT to explore geometrical relationships.

### Probability and statistics

Students distinguish between qualitative or categorical data and quantitative data, and between discrete and continuous data. They understand the concept of a random variable. They can locate sources of bias. They plan questionnaires and surveys to collect meaningful primary data from representative samples. They collect data from secondary sources, including the Internet, and formulate and solve problems related to the data. They group data and plot histograms and other frequency and relative frequency distributions. They calculate measures of central tendency and measures of spread, including variance and standard deviation. They draw stem-and-leaf diagrams and box-and-whisker plots. They plot and interpret simple scatter diagrams between two random variables, and draw a line of best fit where there appears to be correlation. They use relevant statistical functions on a calculator and ICT applications to present statistical tables and graphs.

## Content and assessment weightings for Grade 10

The advanced mathematics standards have four strands: reasoning and problem solving; number, algebra and calculus; geometry and measures; and probability and statistics. Calculus is introduced in Grade 12.

The reasoning and problem solving strand cuts across the other three strands. Reasoning, generalisation and problem solving should be an integral part of the teaching and learning of mathematics in all lessons.

The weightings of the content strands relative to each other are as follows:

Advanced	Number, algebra and calculus	Geometry, measures and trigonometry	Probability and statistics
Grade 10	55%	30%	15%
Grade 11	55%	30%	15%
Grade 12 (quantitative)	40%	–	60%
Grade 12 (for science)	75%	25%	–

The standards are numbered for easy reference. Those in shaded rectangles, e.g. 1.2, are the performance standards for all advanced students. The national tests for advanced mathematics will be based on these standards.

**Many of the Grade 10 advanced standards have been introduced in earlier grades. Teachers should review these standards briefly and devote a greater proportion of time to the work that is new to students.**

## Reasoning and problem solving

By the end of Grade 10, students solve routine and non-routine problems in a range of mathematical and other contexts. They use mathematics to model and predict outcomes of real-world applications. They break down complex problems into smaller tasks, and set up and perform appropriate calculations and manipulations. They identify and use connections between mathematical topics. They develop and explain short chains of logical reasoning, using correct mathematical notation and terms. They generate mathematical proofs and generalise when possible. They approach problems systematically, knowing when to enumerate all outcomes. They conjecture alternative possibilities with ‘What if ...?’ and ‘What if not ...?’ questions. They synthesise, present, interpret and criticise mathematical information, working to expected degrees of accuracy. They recognise when to use ICT and do so efficiently.

### Students should:

#### 1 Use mathematical reasoning to solve problems

- 1.1 Solve routine and non-routine problems in a range of mathematical and other contexts, including open-ended and closed problems.
- 1.2 Use mathematics to model and predict the outcomes of real-world applications.
- 1.3 Identify and use interconnections between mathematical topics.
- 1.4 Break down complex problems into smaller tasks.
- 1.5 Use a range of strategies to solve problems, including working the problem backwards and then redirecting the logic forwards; set up and solve relevant equations and perform appropriate calculations and manipulations; change the viewpoint or mathematical representation, and introduce numerical, algebraic, graphical, geometrical or statistical reasoning as necessary.
- 1.6 Develop short chains of logical reasoning, using correct mathematical notation and terms.
- 1.7 Explain their reasoning, both orally and in writing.
- 1.8 Generate simple mathematical proofs, and identify exceptional cases.
- 1.9 Generalise whenever possible.
- 1.10 Approach a problem systematically, recognising when it is important to enumerate all outcomes.
- 1.11 Conjecture alternative possibilities with ‘What if ...?’ and ‘What if not ...?’ questions.
- 1.12 Synthesise, present, interpret and criticise mathematical information presented in various mathematical forms.

#### Key standards

Key performance standards are shown in shaded rectangles, e.g. 1.2.

#### Cross-references

Standards are referred to using the notation RP for reasoning and problem solving, NA for number and algebra, GM for geometry and measures, and PS for probability and statistics, e.g. standard NA 2.3.

#### Examples of problems

The examples of problems in italics are intended to clarify the standards, not to represent the full range of possible problems.

#### Reasoning and problem solving

Reasoning, generalisation and problem solving should be an integral part of the teaching and learning of mathematics in all lessons.

#### Proofs

Relate to the mathematics in the other strands.

#### Generalisation

Students should appreciate that generalisation is important in mathematics.

1.13 Work to expected degrees of accuracy, and know when an exact solution is appropriate.

1.14 Recognise when to use ICT and when not to, and use it efficiently.

## Number and algebra

By the end of Grade 10, students identify and use number sets and set notation. They calculate with any real numbers, including powers, roots, and numbers expressed in standard form. They use proportional reasoning to solve problems involving scale, ratios and percentages, including compound interest. They are aware of the role of symbols in algebra. They generate and manipulate algebraic expressions, including algebraic fractions, equations and formulae. They multiply any two polynomial expressions and factorise quadratic expressions. They sum arithmetic and geometric sequences and investigate the growth of patterns, generalising relationships to model the behaviour of the pattern. They convert recurring decimals to exact fractions. They use algebraic methods to solve linear and quadratic equations, and a pair of simultaneous linear equations. They plot and interpret straight line and quadratic graphs, and graph regions of linear inequality. They use function notation and find the tangent at a point on the graph of a function. Through their study of linear and quadratic functions and their graphs, and the solution of the related equations, students begin to appreciate numerical and algebraic applications in the real world. They use realistic data and ICT to analyse problems.

### Algebra

Students should learn that algebra enables generalisation and the establishment of relationships between quantities and/or concepts. They should understand the nature and place of algebraic reasoning and proof, and how algebra may be related to geometric concepts, and vice versa.

### Students should:

## 2 Identify and use number sets

2.1 Identify the number sets:

$\mathbb{R}$  the set of all real numbers;

$\mathbb{Z}$  the set of all integers;

$\mathbb{Z}^+$  the set of all positive integers  $\{1, 2, 3, 4, \dots\}$ ;

$\mathbb{Z}^-$  the set of all negative integers;

$\mathbb{Q}$  the set of all rational numbers, i.e. all the different numbers that can be expressed in the form  $a/b$ , where  $a$  and  $b$  are integers with  $b \neq 0$ ;

$\mathbb{N}$  the set of all non-negative integers, called the set of natural numbers  $\{0, 1, 2, 3, 4, \dots\}$ .

*Is  $\mathbb{Z}$  a subset of  $\mathbb{Q}$ ?*

*To what set does  $\sqrt{2}$  belong? How do you know?*

2.2 Know when a real number is irrational, i.e. when it is not a member of  $\mathbb{Q}$ .

2.3 Use and understand the following symbols associated with set theory:  $\mathcal{E}$  for ‘the universal set’;  $\emptyset$  for ‘the null set’;  $\in$  for ‘is a member of’;  $\notin$  for ‘is not a member of’;  $\forall$  for ‘for all’; use brace notation to denote a set.

*$A = \{x: x \in \mathbb{R} \text{ and } 1 \leq x < 10\}$  denotes ‘the set  $A$ , whose members are all real numbers greater than or equal to 1 and less than 10’.*

### Natural numbers

In some texts,  $\mathbb{N}$  is taken to be the same as  $\mathbb{Z}^+$ .

List the elements of each of the following sets:

$A = \{x: x \text{ is a colour of the Qatar flag}\};$

$B = \{x: x \text{ is a state in the GCC}\};$

$C = \{x: x \text{ is a member of the Arab League}\}.$

Is the statement that  $\sqrt[2]{3} \in \mathbb{Q}$  true or false?

What is the solution set of the equation  $x(x+3) = x(x-3) + 6x + 1$ ?

Explain your answer.

- 2.4** Understand the meaning of the *union of two sets A and B* and that this is denoted by  $A \cup B$ , and the meaning of the *intersection of two sets A and B*, denoted by  $A \cap B$ , and represent these sets in a Venn diagram; represent the *complement of set A as A'* and know that  $A \cup A' = \mathcal{E}$ .

Use a Venn diagram to decide whether  $A \cup (B \cap C) = (A \cup B) \cap C$  and  $A \cap (B \cap C) = (A \cap B) \cap C$ .

Describe the set  $\mathbb{Q} \cup \mathbb{Q}'$ .

In a school of 650 students, everyone studies Arabic, English, mathematics and science. They all have to choose to study at least one of art, French, or history. 195 students choose only French. Three times as many students study French and history as study all three subjects, and five times as many study French and art as study all three subjects. 30 students study French and history, 65 do art and history, and 200 do art but not French or history. How many students study history but not art or French?

- 2.5** Know from definitions that every even number can be written in the form  $2m$ , where  $m$  is an integer, and that every odd number can be written in the form  $2n + 1$ , where  $n$  is an integer; understand and use the words *factor*, *multiple*, *divisor*, *prime number*, *prime factor*, *prime factor decomposition*, *least common multiple*, *highest common factor* and *lowest common denominator*.

Prove that the product of two odd numbers is an odd number.

What is the largest prime number you can think of? How do you know it is prime?

Is there a largest prime number? Justify your answer.

What is the highest common factor of  $a^3b^2c$  and  $c^3b^2a$ ?

### 3 Use index notation and solve numerical problems

- 3.1** Understand exponents and  $n$ th roots, and apply the laws of indices to simplify expressions involving exponents; use the  $x^y$  key on a calculator.

Without using a calculator, evaluate  $(5^3)^4 \div 5^{10}$ .

Use a calculator to evaluate  $7^9$ .

Simplify  $8^{1/3} \times 2^{-1}$ .

- 3.2** Know that a root that is irrational is an example of a *surd*, as are expressions containing the addition or subtraction of an irrational root; perform exact calculations with surds.

Calculate  $(\sqrt{2} - 1)(\sqrt{3} - \sqrt{2})$ .

- 3.3** Use standard form in appropriate situations: for exact calculations, to estimate results of calculations and to make comparisons.

To four significant figures, the speed of light is 299 800 000 metres per second. Write this in standard form.

#### Laws of exponents

For  $a > 0$ :

$$a^x \times a^y = a^{x+y}$$

$$a^x \div a^y = a^{x-y}$$

$$(a^x)^y = a^{xy}$$

$$(a^{1/n})^n = a$$

$$a^0 = 1$$

#### Standard form

Use examples from science or geography lessons or real-world applications.

The Earth is approximately a sphere of radius 6378 kilometres. Without using a calculator estimate the circumference at the equator.

The mass of the Earth is  $5.98 \times 10^{24}$  kg. A typical man has a mass of about 70 kg. Approximately how many men would have a total mass equal to that of the Earth?

Light travels at about 300 000 kilometres per second. Use standard form to find the distance away from the Earth of a light-emitting body whose light signal is received at Earth one year after it is emitted.

The Earth completes its orbit around the Sun in 365 days. The Earth is 148.8 million kilometres from the Sun. Assume that the Earth's orbit is circular and that it travels around the Sun with constant speed. Calculate the Earth's speed in kilometres per hour.

Sir Isaac Newton (1642–1727) was a mathematician, physicist and astronomer.

- a. In his work on the gravitational force between two bodies, Newton found that he needed to multiply their masses together.

Work out the value of the mass of the Earth multiplied by the mass of the Moon. Give your answer in standard form.

$$\text{Mass of Earth} = 5.98 \times 10^{24} \text{ kg}$$

$$\text{Mass of Moon} = 7.35 \times 10^{22} \text{ kg}$$

- b. Newton also found that he needed to work out the square of the distance between the two bodies.

Work out the square of the distance between the Earth and the Moon. Give your answer in standard form.

$$\text{Distance between Earth and Moon} = 3.89 \times 10^5 \text{ km}$$

- c. Newton's formula to calculate the gravitational force ( $F$ ) between two bodies is

$$F = \frac{Gm_1m_2}{R^2}$$

where  $G$  is the gravitational constant,  $m_1$  and  $m_2$  are the masses of the two bodies, and  $R$  is the distance between them.

Work out the gravitational force ( $F$ ) between the Sun and the Earth using this formula with the information given below. Give your answer in standard form.

$$m_1m_2 = 1.19 \times 10^{55} \text{ kg}^2$$

$$R^2 = 2.25 \times 10^{16} \text{ km}^2$$

$$G = 6.67 \times 10^{-20}$$

### 3.4 Perform calculations with any real numbers, including mental calculations in appropriate cases.

Calculate mentally the value of  $9999 \times 0.033$ .

Use standard form to estimate the value of  $4350 \times 237.8 \times \pi^2$ .

### 3.5 Add, subtract, multiply and divide any two fractions and understand how to use a unit fraction as a multiplicative inverse.

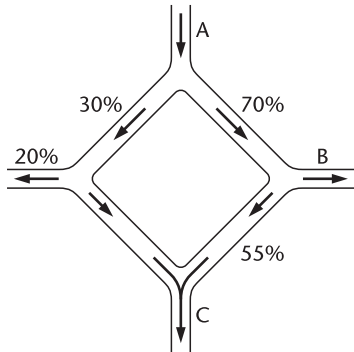
### 3.6 Understand the multiplicative nature of proportional reasoning; form, simplify and compare ratios, and apply these in a range of problems, including mixtures, map scales and enlargements in one, two or three dimensions.

A recipe for six people includes a quarter of a kilogram of figs. How many kilograms of figs would be needed if the recipe were made in the same proportion for eight people?

A map is drawn to scale 1 : 190 000. Two places A and B are 3 cm apart on the map. How far apart are A and B?

**3.7** Perform percentage calculations involving taking a percentage of a percentage, inverse percentage, and compound interest.

The diagram shows water flowing through some pipes. The water starts at A. At each junction the percentage of the inflowing water flowing out through the pipes is indicated. What percentage of the original water flows out at B? What percentage flows out at C?



After Haya's salary is increased by 15% and Abdullah's salary is decreased by 27%, Haya and Abdullah both end up with an annual salary of QR 72 000.

What were their original salaries?

What percentage of Abdullah's original salary was Haya's original salary?

Due to inflation, the price of a television in a store is increased by 15%. In the sales at the end of the year, the price is then reduced by 15%. Does the television revert to its original pre-inflation price? Or is it more, or less? Explain your reasoning.

QR 100 000 has to be invested in deposit accounts. There is a choice of two accounts. One account pays an annual interest of 4.6%. The other account pays interest of 1.5% three times per year. What is the AER of the second account? Which is the better account to invest in and how much more interest will there be after one year in this account than in the other account?

**4 Generate and manipulate algebraic expressions and formulae, and solve algebraic equations**

4.1 Solve any linear equation with one unknown.

**4.2** Generate sequences from term-to-term and position-to-term definitions; investigate the growth of simple patterns, generalising algebraic relationships to model the behaviour of the patterns.

Each term of a sequence is 3 times the preceding term. The first term is 5. Set up a term-to-term definition for this sequence. Give an expression for the  $n$ th term in terms of  $n$ . Write down, but do not simplify, the 50th term.

The table below shows the first six triangular numbers.

Position	1	2	3	4	5	6
Term	1	3	6	10	15	21

Investigate diagrammatic ways of representing these numbers.

Set up a relationship to describe the  $n$ th term in terms of its position value  $n$ . What is the 100th triangular number? What is the 1000th triangular number?

**4.3** Identify and sum arithmetic sequences, including the first  $n$  consecutive positive integers, and give a 'geometric proof' for the formulae for these sums.

**Percentages**

Include real-world financial examples such as taxes, mortgage rates, interest rates, including the annual equivalent rate (AER).

**Generalising**

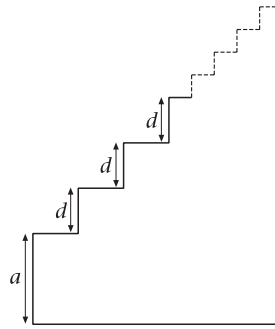
Link algebraic reasoning to geometric concepts where possible.

**ICT opportunity**

Include the use of spreadsheets or a graphics calculator to explore arithmetic sequences.

The diagram is a useful representation of an arithmetic series.

How could you use this diagram to find the sum of the arithmetic series?



Find the sum of the first  $n$  consecutive positive integers, and hence the sum of any set of  $n$  consecutive positive integers.

Find the sum of all numbers between 1 and 100 that are exactly divisible by 3.

- 4.4** Identify and sum geometric sequences and know the conditions under which an infinite geometric series can be summed.

Grains of rice are placed on each square of a chess board. The board has 64 squares. One grain is placed on the first square, two on the second, four on the third, eight on the fourth, and so on. Calculate the total number of grains of rice on the chessboard. Given that 1 kilogram of rice contains approximately 16 000 grains of rice, estimate the weight of all the rice on the chess board.

The sum of the infinite geometric series  $1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + \dots$  is

- A.  $\frac{5}{8}$  B.  $\frac{2}{3}$  C.  $\frac{3}{5}$  D.  $\frac{3}{2}$

Investigate compound interest problems as examples of geometric series.

- 4.5** Convert any recurring decimal to an exact fraction.

Explain why  $0.\dot{1}\dot{2} = \frac{4}{33}$ .

- 4.6** Identify number patterns contained in Pascal's triangle.

Describe carefully in words how any entry in Pascal's triangle is related to entries in the row above. Set up an algebraic relationship to describe this. Where are the triangular numbers located in Pascal's triangle? What other patterns can you spot?

Look at the numbers in an early row of Pascal's triangle. Sum the squares of these numbers. In what row is the answer located? Identify where to find the sum of the squares of the numbers in any row of Pascal's triangle. Explain your reasoning.

- 4.7** Develop confidence and accuracy in working with symbols, understanding that the transformation of all such algebraic objects generalises the well-defined rules of arithmetic. Recognise that letters are used to represent:

- the solution set of initially unknown numbers in *equations*;
- defined variables in *formulae*;
- generalised independent numbers in *identities*;
- new equations, expressions or functions in terms of known, or given, expressions or functions.

Is  $(x + 4)^2 = x(x + 12) - 4(x - 4)$  an equation or an identity? Explain your reasoning.

Is  $(x - a)(x^2 + ax + a^2) = x^3 - a^3$  an equation or an identity? Discuss what happens to this mathematical statement when  $a$  is replaced throughout by  $-a$ .

Give examples of what is meant by an associative law and a distributive law.

Calculate  $(\sqrt{5} + \sqrt{3})(\sqrt{5} - \sqrt{3})$ .

### Geometric sequences

Include consideration of compounding interest more and more frequently.

4.8 Use brackets and correct order of precedence of operations when performing numerical or algebraic calculations.

4.9 Combine numeric or algebraic fractions, and multiply combinations of monomial, binomial and trinomial expressions; multiply any two polynomials, collecting up and simplifying similar terms.

Use Pascal's triangle to identify the coefficients of the powers of  $x$  in the expansion of  $(1+x)^n$  for different values of the positive integer  $n$ . Check the results for  $n=3$  by expanding out  $(1+x)^3$ .

Simplify  $(2x-3)(x^2+x-10)$ .

4.10 Factorise quadratic expressions; conceptualise geometric representations for these factorisations and other similar quadratic expressions.

Without using a calculator, find the exact value of  $7.92^2 - 2.08^2$ .

Explain why  $(a+b)^2 \neq a^2 + b^2$ .

Draw a diagram to represent the identity  $(a+b)^2 = a^2 + 2ab + b^2$ .

Draw a diagram to represent the identity  $(a-b)^2 = a^2 - 2ab + b^2$ .

Construct some quadratic expressions from two linear factors in  $a$  and  $b$  and draw geometric representations for them.

4.11 Solve quadratic equations exactly, by factorisation, by completing the square and by using the quadratic formula.

4.12 Simplify numeric and algebraic fractions by cancelling common factors; rationalise a denominator of a fraction when the denominator contains simple combinations of surds.

Rationalise the expression  $\frac{2}{1+\sqrt{3}}$ .

Simplify the expression  $\frac{a^3b^2 - a^2b^3}{a^2b^2}$ .

4.13 Generate formulae from a physical context, and rearrange formulae connecting two or more variables.

Make  $b$  the subject of the formula  $A = \frac{a+b}{2}$ .

Make  $l$  the subject of the formula  $T = 2\pi\sqrt{\frac{l}{g}}$ .

Make  $x$  the subject of the formula  $w = -z + \frac{x}{v}$ .

Find  $R$  in terms of  $R_1$  and  $R_2$  when  $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$ .

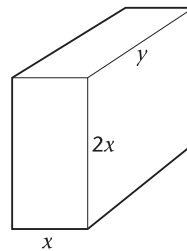
The three different edges of a solid cuboid have lengths  $x$ ,  $2x$  and  $y$ , as shown.

All the lengths are measured in centimetres.

The total surface area of the cuboid is  $800 \text{ cm}^2$ .

Find a formula for  $y$  in terms of  $x$ .

What is the total length of all the edges of the cuboid? Give the answer in terms of  $x$ .



### Quadratic expressions

Include the forms:

$$x^2 + (\alpha + \beta)x + \alpha\beta$$

$$x^2 - (\alpha + \beta)x + \alpha\beta$$

$$x^2 \pm (\alpha - \beta)x - \alpha\beta$$

$$a^2x^2 - b^2y^2$$

Melons cost QR 1.5 each and apples cost QR 3.75 per kilogram. A woman buys apples and melons at the supermarket. Set up a formula to describe the total cost of her purchase. Investigate how many melons and how many kilograms of apples she could buy for QR 30.

The mathematician Johannes Kepler set out three laws of planetary motion in his famous book 'The Harmony of the World', published in 1619. Kepler's third law of planetary motion states that the square of the period of revolution of a planet about the Sun is proportional to the cube of the mean distance of the planet from the Sun. Write this statement as a mathematical equation.

The value of a new car depreciates by 20 per cent at the end of the first year and then loses value at the rate of 10 per cent for every subsequent year. Set up a formula to describe the value  $V$  of the car  $t$  years after purchase. After how many years will the car be worth one quarter of its purchase price?

- 4.14** Substitute an expression for a given variable into a different formula containing this variable.

The volume of a solid cylinder of length  $h$  and radius  $r$  is  $V$ . Find a formula for the curved surface area,  $A$ , of the cylinder in terms of  $r$  and  $h$ . Use this formula to find a formula expressing  $V$  in terms of  $A$  and  $r$ .

## 5 Generate and solve problems with functions and graphs

- 5.1** Investigate a range of mathematical and physical situations to develop the concepts of *function*, *domain* and *range*, recognising one-to-one and many-to-one mappings as functions and a one-to-many mapping as a relation but not a function.

If  $p$  is a person, state with reasons whether each of the following maps are functions:

- $p$  maps to the place of birth of  $p$ ;
- $p$  maps to brother of  $p$ ;
- $p$  maps to nationality of  $p$ ;
- $p$  maps to teacher of  $p$ ;
- $p$  maps to mother of  $p$ .

A firm rents out cars by the day or by the week. The daily charge rate is QR 170 with 150 km free and then QR 2 for every additional kilometre. The weekly charge is QR 1400 with no additional charges. A man needs to hire a car for five days. How many kilometres will he have to drive to make it worthwhile to hire the car for a week?

Look up any country in an atlas and pick six towns from this country. Which of these maps represents a function and which does not: towns  $\rightarrow$  country; country  $\rightarrow$  towns? Justify your answer. What are the domain and range for the mapping that represents a function?

- 5.2** Understand and use the concept of related variables and, in special cases, set up appropriate functional relationships between them.

In an electric circuit,  $V = IR$ , where  $V$  is the voltage in volts,  $I$  is the current in amps and  $R$  is the resistance in ohms. The electrical power in watts is  $P = VI$ . Find a formula connecting the variables  $P$ ,  $V$  and  $R$ .

- 5.3** Plot a graph to show the relationship between two variables given quantitative information between the variables in tabular or algebraic form.

- 5.4** Use a graphics calculator or graph plotter and pencil and paper methods to plot and interpret a range of functional relationships, some continuous and others discontinuous, arising in familiar contexts.

### Functions

Include use of the notation  $y = f(x)$  to denote that  $y$  is a function of  $x$ .

### Functional relationships

Include examples drawn from science.

### ICT opportunity

Graphics calculators can be used to explore a range of functional relationships.

Draw a graph showing the functional relationship between postage rate in Qatar and the weight of package to be posted.

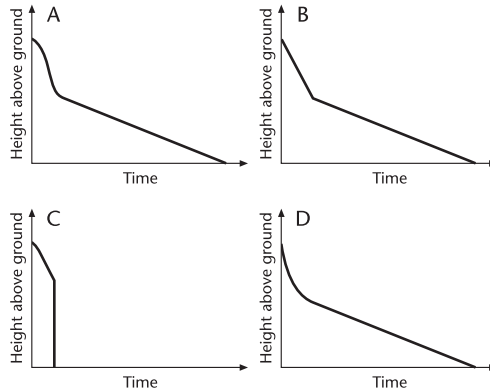
Plot the graph of  $y = 1/x^2$  for the domain set  $\{x: x \in \mathbb{R} \text{ and } 1 \leq x \leq 4\}$ .

Discuss whether the domain could be extended.

Plot the curve  $y = \sqrt{x}$  on a suitably defined domain. Discuss why the domain cannot be the set  $\mathbb{R}$ . Compare this curve with the curve of  $y = x^2$ , drawn on the same axes.

Ahmed does a parachute jump. He jumps out of the plane and falls faster and faster towards the ground. After a few seconds his parachute opens. He slows down and then falls to the ground at a steady speed.

Which of these graphs shows Ahmed's parachute jump? Explain why each of the other graphs is wrong.



Invent examples of functions with different definitions on different subdomains, for example, electricity charges as a function of the number of units of electricity used.

The Int  $x$  function, written as  $[x]$ , maps  $x$  to the greatest integer less than or equal to  $x$ . Find  $[5.9]$ ,  $[6]$  and  $[-4.7]$ .

Plot on the same axes the curves  $y = 2^x$  and  $y = 2^{-x}$  for  $-3 \leq x \leq 3$ . Describe the features of the two curves. Discuss situations that could be modelled by these equations.

A rectangular enclosure has a wall on one side and the other three sides are made of metal fencing. The side parallel to the wall has length  $d$ , measured in metres. The enclosure has an area of  $600 \text{ m}^2$ . Show that the total length,  $L$  metres, of fencing is given by  $L = d + 1200/d$ . Plot this function using a graphics calculator. Find from the graph the value of  $d$  that makes  $L$  as small as possible.

**5.5** Recognise when a graph represents a functional relationship between two variables and when it does not.

Discuss whether or not the graph of i. a circle and ii. a semicircle represents a function. Look at any special cases that may arise.

### Direct proportion

**5.6** Translate the statement  $y$  is proportional to  $x$  into the symbolism  $y \propto x$  and into the equation  $y = kx$ , and know that the graph of this equation is a straight line through the origin and that the constant of proportionality,  $k$ , is the gradient of this line.

**5.7** Know that if two coordinate variables are connected by a straight line graph that passes through the origin of coordinates, then each coordinate variable is proportional to the other; use relevant information to find  $k$ .

**5.8** Identify common examples of two linear quantities varying in direct proportion to each other.

### ICT opportunity

Graphics calculators or graph plotters can be used to explore a range of functional relationships.

### Direct proportion

Use examples from science and the real world, e.g.

- conversion rates;
- Ohm's law, or  $V = IR$ ;
- the pressure of gas in a constant volume is directly proportional to its temperature.

## Straight lines and linear functions

**5.9** Know that a straight line in the explicit form  $y = mx + c$  represents a function, but that a straight line in the implicit form  $ax + by + d = 0$  may, or may not, be a function; know that any straight line in the  $xy$ -plane can be represented in this implicit form, but that only certain lines in the plane can be represented by the explicit form; work with both of these forms.

**5.10** Plot the graphs of the equations in 5.9; know the meanings of *gradient of the line* and *intercept on the  $x$ - or  $y$ -axis* and relate these to the coefficients  $a$ ,  $b$  and  $d$ , or to the coefficients  $m$  and  $c$ .

*What is the gradient of the line  $3x + 2y - 5 = 0$ ?*

*Find an equation of a line that is perpendicular to this line.*

*Draw the two lines on a graph.*

*A triangle has its vertices at the points  $(1, 3)$ ,  $(2, 5)$  and  $(3, 4.5)$ .*

*Find the equations of the lines containing each side.*

*Is the triangle a right-angled triangle? Explain how you know.*

*What angle does the line  $y = \sqrt{3}x + 1$  make with the positive  $x$ -axis?*

**5.11** Construct the Cartesian equation of a straight line from its graph alone, or from the knowledge of the coordinates of two points on the line, or from the coordinates of one point on the line and the gradient of the line.

*What is the equation of the straight line through the points  $(5, -2)$  and  $(-4, 3)$ ? What is the gradient of this line? Where does it cross the  $y$ -axis? Where does it cross the  $x$ -axis?*

*A triangle has vertices at the points  $(1, 1)$ ,  $(5, -4)$  and  $(-3, 2)$ . Find the equation of each of its sides.*

**5.12** Know the condition for two straight lines to be parallel or perpendicular, including the special cases of one of the lines being parallel to either axis.

*Give equations of lines parallel and perpendicular to the line  $y = 5x - 3$ .*

**5.13** Read off the coordinates of the point of intersection, given the graphs of two intersecting straight lines; find exactly, by algebraic means, the coordinates of the point of intersection of two lines, given their equations.

*Find the intersection point of the line  $y = 4x + 2$  with the line  $y = 9 - 3x$ .*

*Discuss whether two lines have no intersection, a unique intersection point, or infinitely many intersection points.*

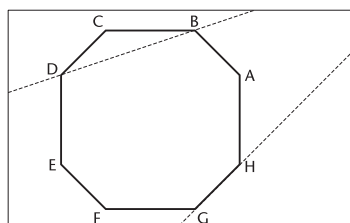
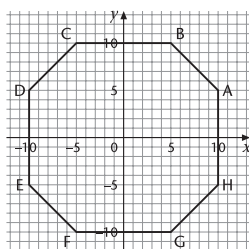
*In two dimensions discuss whether non-parallel lines must intersect.*

*What happens in three dimensions?*

**5.14** Interpret the solution set of the simultaneous equations  $E_1$  and  $E_2$ , where  $E_1$  and  $E_2$  are the equations of two straight lines.

*Look at this octagon. The line through  $D$  and  $B$  has the equation  $3y = x + 25$ .*

*The line through  $G$  and  $H$  has the equation  $x = y + 15$ .*



## Gradients

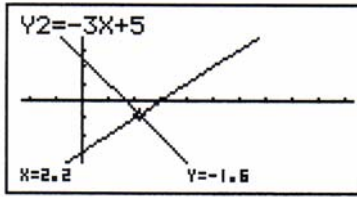
Include the terms *slope* and *rate of change*.

Solve the simultaneous equations

$$y = 2x - 6$$

$$y + 3x = 5$$

to find the point of intersection of these two lines.



Here are the equations of some straight lines:

$$y = 2x - 7; \quad y = 7 - 2x; \quad y = 2x + 9; \quad y = 14 - 4x;$$

$$y = -10; \quad y = -10 + 2x; \quad x = 1; \quad y = -0.5x + 8.$$

List all the pairs of lines that are: a. parallel to each other; b. perpendicular to each other; c. different representations of the same line. From the lists, find pairs of lines that intersect in a unique point and find the intersection point in each case.

- 5.15** Draw the tangent line at a point on the graph of a function, calculate the slope of this line and interpret the behaviour of the function at that point, knowing whether the function is increasing or decreasing at the point, or stationary.

### Direct proportion (continued)

- 5.16** Translate the statement  $y$  is proportional to  $x^2$  into the symbolism  $y \propto x^2$  and into the equation  $y = kx^2$  and know that the graph of this equation is a parabola through the origin.

A body falling from rest under the force of gravity falls a distance  $s$  metres in time  $t$  seconds where  $s = 4.9t^2$ . Find the distance fallen after 5 seconds. How long does it take the body to fall 30 metres?

Discuss how to plot a linear graph  $s = 4.9z$ , by defining the variable  $z = t^2$ .

- 5.17** Identify some other common examples of proportional variation.

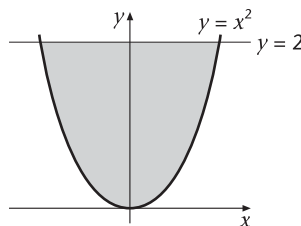
### Linear and quadratic inequalities

- 5.18** Graph regions of linear inequality and solve simple problems (e.g. elementary linear programming) represented by such regions; understand simple quadratic inequalities.

A firm delivers new cars to Doha. It has a contract to deliver at least 65 cars each day. The firm owns 7 carriers that can each carry 8 cars and 5 carriers that can each carry 10 cars. The firm employs 8 drivers and each carrier can only make one journey with a full load each day. What is the maximum numbers of cars that can be delivered each day? What is the minimum number of drivers needed to fulfil the contract?

The shaded region is bounded by the curve  $y = x^2$  and the line  $y = 2$ .

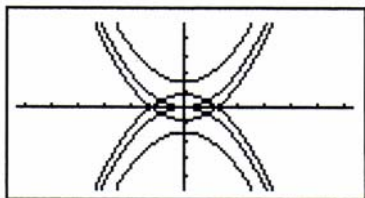
What two inequalities together fully describe the shaded region?



## Quadratic functions

- 5.19** Recognise a second-order polynomial in one variable,  $y = ax^2 + bx + c$ , as a quadratic function; plot graphs of such functions (recognising that these are all parabolas) and identify the intercepts with the coordinate axes, the axis of symmetry and the coordinates of the maximum or minimum point; understand when quadratic functions are increasing, when they are decreasing and when they are stationary.

*Create a display like this with your graphics calculator.*



- 5.20** Model a range of situations with quadratic functions of the form  $y = ax^2 + c$ .

## Geometry and measures

By the end of Grade 10, students use their knowledge of geometry, Pythagoras' theorem and the trigonometry of right-angled triangles to solve practical and theoretical problems relating to shape and space. They understand congruence and similarity. They prove that the perpendicular from the centre of a circle to a chord bisects the chord and that the two tangents from an external point to a circle are of equal length. They carry out straight edge and compass constructions and determine the locus of an object moving according to a rule. They use radians as a measure of angle, and dimensionally correct units for length, area and volume. They solve problems involving rates and compound measures. They use formulae to calculate the length of an arc and the area of a sector of a circle, the area of any triangle, trapezium, parallelogram or quadrilateral with perpendicular diagonals, and the surface area and volume of a right prism, cylinder, cone, sphere and pyramid. They use bearings, latitude, longitude and great circles to solve problems relating to position, distance and displacement on the Earth's surface. They use ICT to explore geometrical relationships.

### Students should:

#### 6 Develop geometrical reasoning and proof, and solve geometric problems

##### Congruence and similarity: properties of angles, straight lines and triangles

- 6.1** Use knowledge of angles at a point, angles on a straight line, and alternate and corresponding angles between parallel lines and a transversal line to present formal arguments to establish the congruency of two triangles.

*Prove that each of the angles in an equilateral triangle is  $60^\circ$ .*

### ICT opportunity

Include the use of a graphics calculator or graph plotter.

### Geometry and measures

Students should develop an appreciation of the importance and range of geometrical applications in the real world, and the aesthetic qualities of geometric models. They should understand the nature and place of geometric reasoning and proof, and how geometry may be related to algebraic concepts, and vice versa.

### Use of ICT

Geometry is enhanced with use of a dynamic geometry system, or DGS, which provides an interactive focus to investigate and conjecture results which could then be proved as theorems.

**6.2** Establish the congruency of two triangles to generate further knowledge and theorems about triangles, including proving that the base angles of an isosceles triangle are equal and that the line joining the mid-points of two sides of a triangle is parallel to the remaining side.

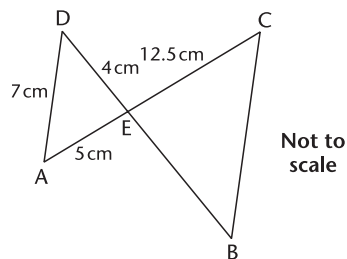
**6.3** Understand similarity of two triangles and other rectilinear shapes, knowing that similarity preserves shape and angles, but not size; make inferences about the lengths of sides and about the areas of similar figures; prove that if two triangles are similar, then the ratio of the areas of the two triangles is the square of the ratio of any pair of corresponding sides of the two triangles chosen in the same order; in three dimensions, calculate the ratio of the volume of a scale model to the volume of the actual object.

*A goldsmith has a block of gold in the shape of a cube. He wants to make another gold cube that has exactly twice the volume of the first cube. What scale factor must he use?*

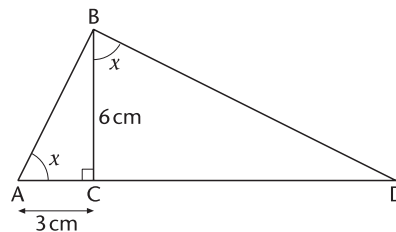
*Two similar shaped gas-filled balloons are made of a special material. The area of material used in one balloon is  $100 \text{ cm}^2$ . The material for the other balloon has an area of  $225 \text{ cm}^2$ . Calculate the ratio of the volume of the larger balloon to the volume of the smaller balloon. Give this ratio in its simplest form.*

*The diagram shows two triangles ADE and BCE. Side AD is parallel to side BC. Explain why the two triangles are similar to each other.*

*Calculate the missing lengths for triangle BCE.*



*Calculate the length of CD in the diagram on the right.*



*A scale model of a dhow has a volume of  $300 \text{ cm}^3$ . The length of the actual dhow is 100 times longer than the length of the model. What is the volume of the dhow? Give the answer in appropriate units.*

**6.4** Calculate the interior and exterior angles of regular polygons; name polygons with up to ten sides.

**Trigonometry, Pythagoras' theorem and the solution of triangles**

**6.5** Know the standard trigonometric ratios, and their standard abbreviations, for sine of  $\theta$ , cosine of  $\theta$  and tangent of  $\theta$ , given an angle  $\theta$  in a right-angled triangle, and use these ratios to find the remaining sides of a right-angled triangle given one side and one angle or to find the angles given two sides.

*Show that  $\tan \theta = \frac{\sin \theta}{\cos \theta}$ .*

**6.6** Derive and recall the exact values for the sine, cosine and tangents of  $0^\circ$ ,  $30^\circ$ ,  $45^\circ$ ,  $60^\circ$ ,  $90^\circ$  and use these in relevant calculations.

*Calculate the exact area of an equilateral triangle with sides of length 6 cm.*

**Trigonometric ratios**

Use a calculator to find sine and cosine values of a given angle and to find the angle corresponding to a given value of the sine or cosine of that angle.

Students should know that these are inverse functions defined on a restricted domain.

Show that  $\cos^2 60^\circ + \sin^2 60^\circ = 1$ . Show that a similar result is true when the angle is replaced by any of  $0^\circ, 30^\circ, 45^\circ, 90^\circ$ .

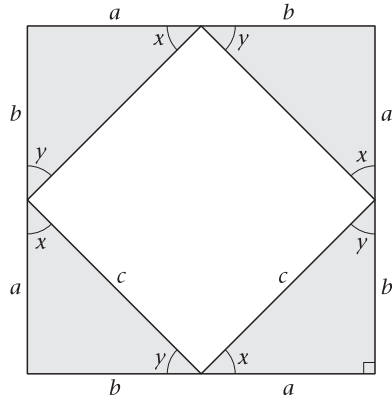
On a particular day, the depth of the water,  $d$  metres, in a harbour is given by:

$$d = 7 + 2 \sin(30t)^\circ$$

where  $t$  is the time, in hours, since midnight. What is the maximum depth of water? At what time is the depth of the water at its maximum?

**6.7** Discuss at least two proofs of Pythagoras' theorem.

Explain how you could use the diagram below to prove Pythagoras' theorem.



**6.8** Use Pythagoras' theorem to find the distance between two points, to solve triangles, to find Pythagorean triples, and to set up the Cartesian equation of a circle of radius  $r$ , centred at the point  $(\alpha, \beta)$ .

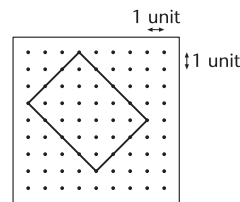
Find the equation of a circle of radius 5 units, centred at the point  $(5, -3)$ .

Find the exact distance between the point  $(1, 4)$  and the point  $(-2, 5)$ .

Two sides of a right-angled triangle are of length 21 cm and 29 cm. What are the possible lengths of the remaining side?

In this question, you should not use a calculator.

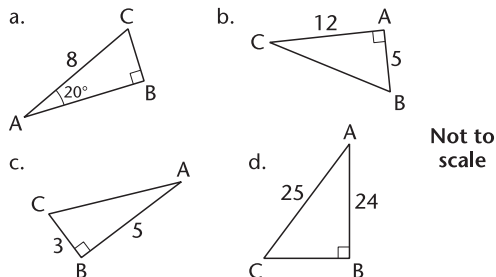
An elastic band is fixed on four pins on a pinboard, as shown in the diagram.



Show that the total length of the band in this position is  $14\sqrt{2}$  units.

Show that a triangle with sides of length  $m^2 - n^2$ ,  $2mn$  and  $m^2 + n^2$  respectively is always right-angled. Find some right-angled triangles using this result.

Solve the triangles shown, giving all the angles and all the sides.



Each side of a cube is 5 cm. Calculate the length of a diagonal of the cube from one vertex on the 'base' to the opposite vertex on the 'top face'. What is the angle between this diagonal and the base?

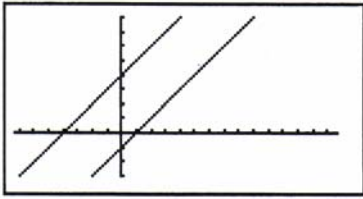
**Powers of (co)sines**

Note that  $(\cos \theta^\circ)^2$  is written as  $\cos^2 \theta^\circ$  and that  $(\sin \theta^\circ)^2$  is written as  $\sin^2 \theta^\circ$ .

**Pythagoras' theorem**

There are many interesting websites devoted to proofs and applications of this fundamental theorem in geometry.

Suggest possible equations for these straight lines.  
Find the shortest distance between them.



### Circle theorems

6.9

Prove the circle theorems:

- The perpendicular from the centre of a circle to a chord bisects the chord.
- The two tangents from an external point to a circle are of equal length.

### Circle theorems

Include terms associated with a circle: *centre, radius, diameter, circumference, arc length, sector, segment, chord, tangent.*

Include the use of a dynamic geometry system (DGS).

### Constructions

6.10

Perform and justify straight edge and compass constructions, including those to bisect a line, to construct an equilateral triangle with a side of given length, to drop a perpendicular from a point to a line, and to bisect an angle.

*Construct a square. You may only use a straight edge, a pencil and a pair of compasses.*

*Use the construction to bisect an angle several times over to construct an angle of  $22.5^\circ$ .*

*Explain why the construction to bisect an angle works.*

### Loci

6.11

Determine the locus of an object moving according to a rule, including those arising in simple physical situations.

*A goat is on a rope attached at one corner of a rectangular enclosure. The enclosure measures 10 m by 4 m. The rope is 6 m long. Draw a scale drawing of the enclosure and shade in the locus in which the goat can move.*

*Find the locus of all points 3 cm from a circle of radius 5 cm. Discuss how the locus is changed if three dimensions are allowed.*

### Transformations

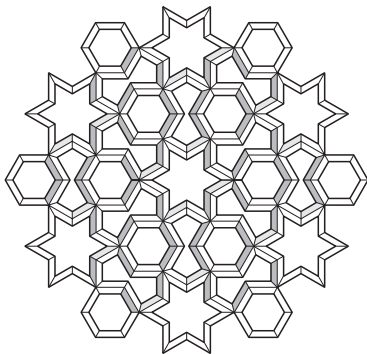
6.12

Investigate Islamic patterns and describe their features.

*This pattern is from a mosque in Isfahan, in Iran. Use this and other Islamic patterns to discuss key features of the pattern (its construction, reflection symmetries, translations, and so on).*

### Transformations

Transformations are best developed through use of DGS.



### Use of ICT

6.13

Use ICT to explore geometrical relationships.

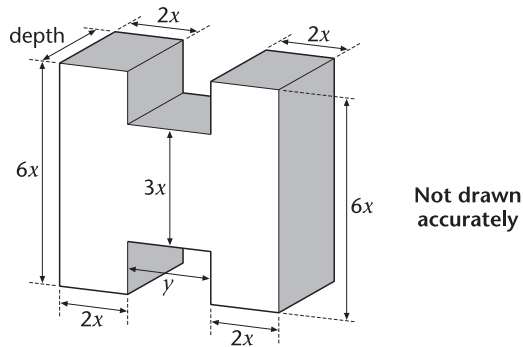
## 7 Use a range of measures and compound measures to solve problems

- 7.1** Find perimeters and areas of rectilinear shapes and volumes of rectilinear solids; find the circumference and area of a circular region, and the surface area and volume of a right prism, cylinder, cone and pyramid, and a sphere, using dimensionally correct units.

The volume of a pyramid is (base area  $\times$  perpendicular height).

Calculate the volume of a pyramid with a square base with side 4 cm and a volume of  $48 \text{ cm}^3$ . What is its perpendicular height?

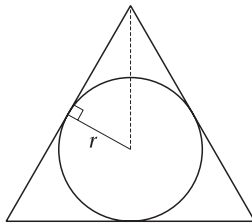
This prism was made from three cuboids.



Show that the area of the cross-section of the prism is  $24x^2 + 3xy$ .

The volume of the prism is  $3x^2(8x + y)$ . What is the depth of the prism?

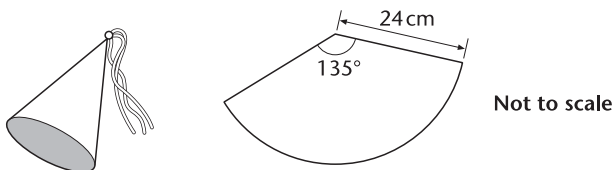
A round peg, of radius  $r$ , just fits into an equilateral triangular hole.



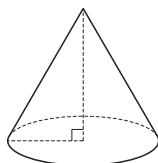
What proportion of the hole is filled by the peg?

- 7.2** Use radians to calculate sector areas and arc lengths.

A manufacturer makes party hats shaped like hollow cones. To make the hats she cuts pieces of card which are sectors of a circle, radius 24 cm. The angle of the sector is  $135^\circ$ .



- Show that the arc length of the sector is  $18\pi \text{ cm}$ .
- The sector is joined edge to edge to make a cone. The edges of the sector meet exactly with no overlap. Calculate the vertical height of the completed hat.



### Sectors and arcs

Include terms associated with a circle: centre, radius, diameter, circumference, arc length, sector, segment, chord.

Define 1 radian as the angle that an arc of length 1 unit subtends at the centre of a circle of radius 1 unit.

A satellite is 1500 km above the Earth. It has a camera with a  $50^\circ$  angle of view with which it surveys the Earth below. Draw a diagram to represent the satellite and its camera in relation to the Earth. Calculate how far apart the two furthest points on the Earth are that can be photographed by the satellite at any one time. Take the Earth to be a sphere of radius 6378 kilometres.

**7.3** Use bearings, latitude, longitude and great circles to solve problems relating to position, distance and displacement on the Earth's surface.

*How would you find the shortest distance between Doha and London?*

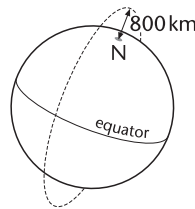
*A plane flies from Doha to Karachi almost along the line of latitude 25 degrees north. Doha is at longitude 51 degrees east approximately and Karachi is at longitude 67 degrees east approximately. How far is it from Doha to Karachi along this route?*

*What is a great circle of the Earth?*

*An oil tanker sails 350 km from Doha towards Dubai on a bearing of  $090^\circ$  and then from Dubai towards Al Kuwait on a bearing  $310^\circ$ . Al Kuwait is about 600 km from Doha. Approximately, how far is it from Dubai to Al Kuwait?*

**7.4** Solve problems involving compound measures, including average speed, such as cost per litre, kilometres per litre, litres per kilometre, population density (number of people per unit area), density (mass per unit volume), pressure (force per unit area) and power (energy per unit time).

*A satellite passes over both the north and south poles, and it travels 800 km above the surface of the Earth. The satellite takes 100 minutes to complete one orbit.*

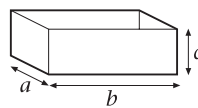


*Assume the Earth is a sphere and that the diameter of the Earth is 12 800 km. Calculate the speed of the satellite, in kilometres per hour.*

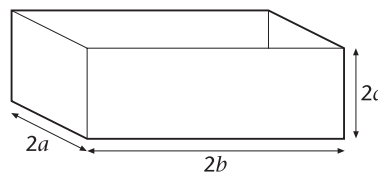
*A water tank is filled through a hosepipe connected to a tap. The rate of flow through the hosepipe can be varied. A tank of capacity 4000 litres fills at a rate of 12.5 litres per minute. How long in hours and minutes does it take to fill the tank?*

*Another tank takes 5 hours to fill at a different rate of flow. How long would it take to fill this tank if this rate of flow is increased by 100%? How long would it have taken to fill this tank if the rate of flow had been increased by only 50%?*

*This tank, measuring  $a$  by  $b$  by  $c$ , takes 1 hour 15 minutes to fill.*



*How long does it take to fill  $2a$  by  $2b$  by  $2c$ , at the same rate of flow?*



### Compound measures

Use appropriate units and dimensions. Stress how units are calculated in compound measures.

Draw on examples from science.

# Probability and statistics

By the end of Grade 10, students distinguish between qualitative or categorical data and quantitative data, and between discrete and continuous data. They understand the concept of a random variable. They can locate sources of bias. They plan questionnaires and surveys to collect meaningful primary data from representative samples. They collect data from secondary sources, including the Internet, and formulate and solve problems related to the data. They group data and plot histograms and other frequency and relative frequency distributions. They calculate measures of central tendency and measures of spread, including variance and standard deviation. They draw stem-and-leaf diagrams and box-and-whisker plots. They plot and interpret simple scatter diagrams between two random variables, and draw a line of best fit where there appears to be correlation. They use relevant statistical functions on a calculator and ICT applications to present statistical tables and graphs.

## Probability and statistics

Students should know that statistics is the branch of mathematics used to predict the outcomes of large numbers of events when these outcomes are uncertain, and that probability lies at the heart of statistics. They should be aware of the uses of statistics in society and recognise when statistics are used sensibly and when they are misused or likely to be misunderstood.

## Students should:

### 8 Collect, process, represent, analyse and interpret data and reach conclusions

#### Sampling

##### 8.1 Know that:

- it is important to choose representative samples;
- in a random sample there are chance variations;
- in a biased sample there are systematic differences between the sample and the population from which it is drawn.

##### 8.2 Locate obvious sources of bias within a sample.

##### 8.3 Know that different types of data can be collected from samples – qualitative/categorical data (e.g. eye colour, male, female) and quantitative data (e.g. age, height, lifespan, mortality rates) – and that quantitative data may be discrete (e.g. number of defective items in a production process) or continuous (e.g. weight); understand the concept of a random variable.

#### Statistical techniques

##### 8.4 Plan surveys and design questionnaires to collect meaningful primary data from representative samples in order to test hypotheses about, or estimate, characteristics of the population as a whole.

##### 8.5 Formulate and solve problems using secondary data from published sources, including the Internet.

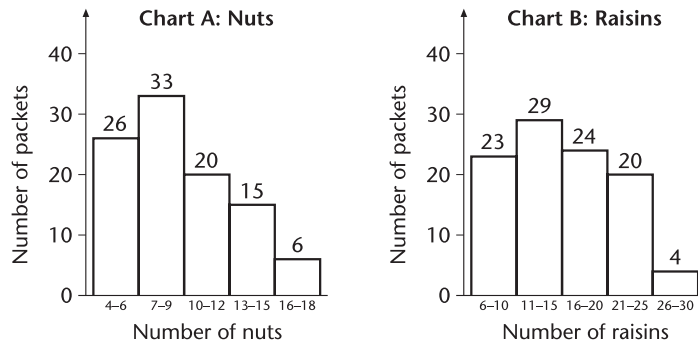
##### 8.6 In analysing data, calculate and use measures of central tendency such as the arithmetic mean and the median.

*Investigate life expectancy in a range of countries, including Qatar, Iran, Turkey, India, Brazil, China, Russia, Italy, the United Kingdom and the United States of America.*

## Data collection

Include data collected in other subjects, such as science or social science.

A company makes breakfast cereal containing nuts and raisins. They counted the number of nuts and raisins in 100 small packets.



- Calculate an estimate of the mean number of nuts in a packet.
- Calculate an estimate of the number of packets that contain 24 or more raisins.

**8.7** Calculate measures of spread, including the variance and standard deviation.

A fisherman kept a record of the mass,  $M$ , of each of the fish he caught in one season.

Mass (kg)	Frequency
$0.5 < M \leq 1$	14
$1 < M \leq 1.5$	29
$1.5 < M \leq 3$	45
$3 < M \leq 4$	16
$4 < M \leq 6$	10
Total 114	

The fisherman exaggerated the mass of fish caught. He added 0.5 kg to the mass of each fish before he recorded it. State what effect this would have on the estimate of the mean. State what effect this would have on the estimate of the standard deviation.

Another fisherman doubled the mass of each fish before he recorded it. Comment on the effect this would have on the mean and standard deviation of the mass of fish caught.

Find the mean and median salaries of the group of workers in Qatar whose weekly salaries in riyals are given in the table below.

Salary (QR)	250	300	350	400	450	500	550	600
Frequency	5	11	20	31	18	12	7	3

Which average is the most representative for these workers? Justify your answer.

Use statistical functions on a calculator to calculate the standard deviation for the salaries in this group. What information does this convey?

**8.8** Construct histograms, grouping continuous data when necessary.

A scientist wanted to investigate the lengths of eggs from a particular breed of hen. Taking a sample of 80 eggs, she measured the length of each one and grouped the data as follows:

Length ( $l$ ) in cm	$4.4 \leq l < 5.0$	$5.0 \leq l < 5.4$	$5.4 \leq l < 5.8$	$5.8 \leq l < 6.3$	$6.3 \leq l < 6.5$
Frequency	4	20	36	16	4

Complete a histogram to show this information.

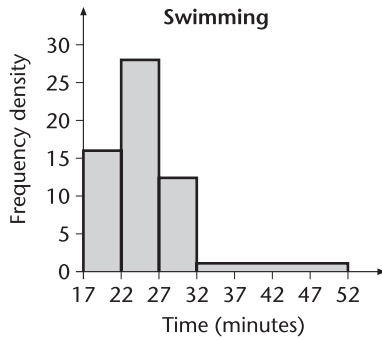
Write the frequency density on each part of the histogram.

Calculate the mean length of the eggs in her sample. Discuss how to calculate best estimates for the modal and median values of the lengths of the eggs in the sample.

**Histograms**

Include the terms frequency, frequency distribution and frequency density, relative frequency and relative frequency distribution.

304 people took part in a swimming contest. They swam 1.5 km. The histogram shows the distribution of their times for the event.



a. The histogram is constructed using frequency densities. The table shows the frequency densities. Complete the table to show the frequencies.

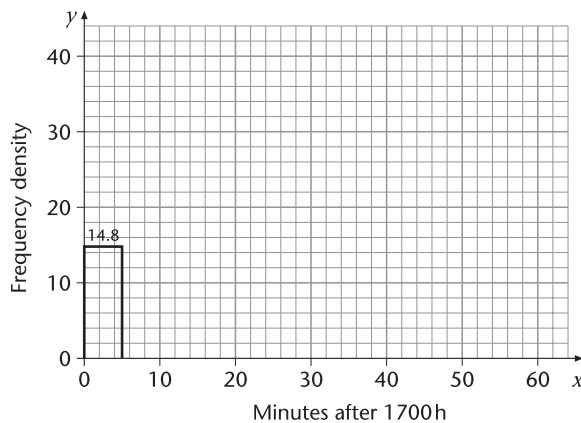
Time $t$ (minutes)	Frequency density	Frequency
$17 \leq t < 22$	16.0	80
$22 \leq t < 27$	28.0	
$27 \leq t < 32$	12.4	
$32 \leq t < 52$	1.1	

- b. 304 people took part. Calculate an estimate of the mean time for this event.  
 c. Explain why the median time for the event must be between 22 and 27 minutes.  
 d. Calculate an estimate of the median time for this event.

The table below shows the number of cars leaving a car park during the periods given.

Number of minutes after 1700 h	$0 \leq n < 5$	$5 \leq n < 10$	$10 \leq n < 20$	$20 \leq n < 50$	$50 \leq n < 60$
Number of cars leaving	74	115	248	1174	189

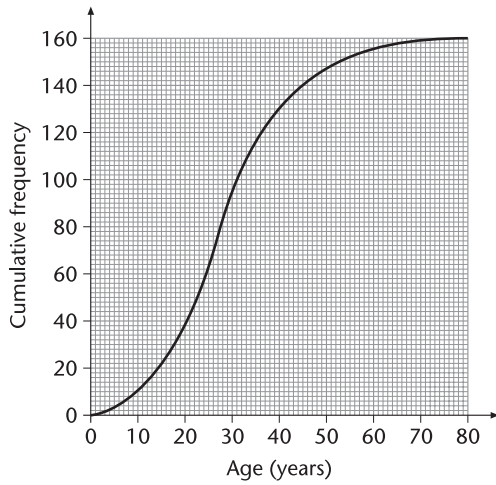
Complete the histogram to show the information in the table.  
 Write the frequency density above each rectangle of the histogram.



The value 14.8 on the histogram is the frequency density for the period  $0 \leq n < 5$  minutes. Explain what is meant by frequency density with regard to cars leaving the car park.

**8.9** Plot cumulative frequency distributions, grouping continuous data when necessary.

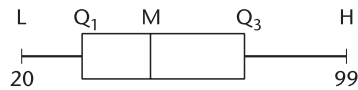
Sulaiman did a survey of the age distribution of 160 people at a theme park. The cumulative frequency graph below shows his results.



- Use the graph to estimate the median age of the 160 people at the theme park.
- Use the graph to estimate the interquartile range of the age of the 160 people at the theme park.

**8.10** Draw stem-and-leaf diagrams and box-and-whisker plots and use them in presentations of findings.

The diagram shows a box-and-whisker plot of examination marks for a class of students.



$L$  represents the lowest mark scored and  $H$  is the highest mark scored.  $LH$  then represents the range of marks.  $Q_1$  is the first quartile mark,  $Q_3$  is the third quartile mark and  $Q_1Q_3$  is the interquartile range.  $M$  is the median mark.

A school for boys and a school for girls each enter students for the same mathematics examination. The girls' marks were:  
 97 98 57 45 63 75 87 34 56 28 67 89 45 61 53 49 81 32 23 45 47 72 34 54 23 100 76 47.  
 The boys' marks were:  
 67 87 83 92 34 31 23 25 29 39 89 91 54 47 41 50 77 18 89 10 26 62 39 14 90.

Draw back-to-back stem-and-leaf diagrams to represent these scores. Compare the performances of the girls and the boys, explaining your methodology and findings.

Using the above data, plot a cumulative frequency graph for the marks of the girls. What was the median score?

What was the interquartile range of the distribution of marks?

Draw a box-and-whisker plot to represent the girls' marks.

Draw a relative frequency histogram for these data, explaining how the data were grouped and the meaning of each bar of the histogram.

**8.11** Make inferences and draw conclusions from the formulation of a problem to the collection and analysis of data in a range of situations; select statistics and a range of charts, graphs and tables to present findings.

Compare the television viewing habits of students in different grades at school.

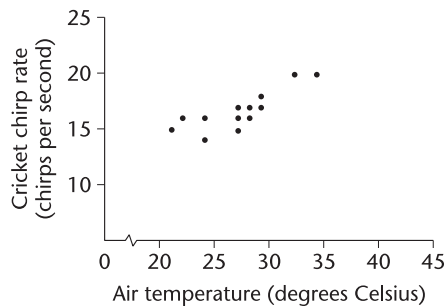
**Cumulative frequency**  
 Include the terms *range*, *percentile*, *interquartile range*, *semi-interquartile range*, and *mode*, *modal class*, *modal frequency*.

## 9 Simple correlation

- 9.1** Draw scatter diagrams between two random variables associated with some common context; identify through elementary qualitative discussion positive and negative correlation; where there appears to be correlation, draw a line of best fit, constructing it to pass through the point representing the arithmetic means of the two variables in the chosen samples, judging by eye the line about which the data points are most evenly distributed.

*Compare the examination marks for all students in a class for: a. mathematics and science; b. Arabic and English; c. mathematics and art. Discuss whether there appears to be correlation or not.*

*Scientists have observed that insects called crickets move their wings faster in warm temperatures than in cold temperatures. Below is a graph showing 13 observations of cricket 'chirps' per second and the associated air temperatures.*



- On the graph, draw the estimated line of best fit for this data.*
- Using your line, estimate the air temperature when cricket chirps of 22 per second are heard.*

**TIMSS Grade 12**

## 10 Use of ICT

- 10.1** Use a calculator with statistical functions to aid the analysis of large data sets, and ICT packages to present statistical tables and graphs.

### ICT opportunity

A range of applications can be used to support data handling.