Mathematics standards

Grade 10
Foundation

Summary of students' performance by the end of Grade 10

Reasoning and problem solving

Students solve routine and non-routine problems in a range of mathematical and other contexts, and use mathematics to model and predict the outcomes of real-world applications. They identify and use connections between mathematical topics. They break down complex problems into smaller tasks, and set up and perform appropriate manipulations and calculations. They develop and explain short chains of logical reasoning, using correct mathematical notation and terms. They generate simple mathematical proofs and identify exceptional cases. They aim to generalise. They approach problems systematically, knowing when it is important to enumerate all outcomes. They conjecture alternative possibilities with 'What if …?' and 'What if not …?' questions. They synthesise, present, interpret and criticise mathematical information, working to expected degrees of accuracy. They recognise when to use ICT and do so efficiently.

Number and algebra

Students identify and use number sets and set notation. They calculate with any real numbers, including powers, roots, and numbers expressed in standard form. They use proportional reasoning to solve a range of problems involving scale, ratios and percentages. They are aware of the role of symbols in algebra. They generate and manipulate algebraic expressions, including algebraic fractions, equations and formulae. They sum arithmetic sequences and investigate the growth of simple patterns, generalising relationships to model the behaviour of the patterns. They find the solution of any linear equation, and a pair of simultaneous linear equations, and plot straight line and simple quadratic graphs. They use function notation. Through their study of linear and simple quadratic functions and their graphs, and the solution of the related equations, students begin to appreciate numerical and algebraic applications in the real world. They use realistic data and ICT to analyse problems.

Geometry and measures

Students use their knowledge of geometry, Pythagoras' theorem and the trigonometry of right-angled triangles to solve practical and theoretical problems relating to shape and space. They understand congruence and similarity. They carry out straight edge and compass constructions and determine the locus of an object moving according to a rule. They use a range of SI units and measures, including bearings and compound measures. They use formulae to calculate: the circumference and area of a circle; the perimeter and area of any triangle, or trapezium, parallelogram or quadrilateral with perpendicular diagonals; the surface area and volume of a right prism, cylinder, square-based pyramid and cone; and the volume of a sphere. They use ICT to explore pattern, similarity, congruence and constructions.

Probability and statistics

Students know that statistical data are collected from observation or measurement on samples taken from a larger population, and that, by analysing the sample data, inferences can be made about the population as a whole. They distinguish between qualitative (or categorical) data and quantitative data, and between discrete and continuous data. They plan simple surveys, design simple questionnaires and plot histograms in which the height of each bar is proportional to the frequency of that class. They calculate means and medians, and understand mode and modal class. They plot and interpret simple scatter diagrams between two random variables, and draw a line of best fit where there appears to be correlation. They use relevant statistical functions on a calculator and ICT applications to present statistical tables and graphs.

Content and assessment weightings for Grade 10

The foundation mathematics standards are grouped into four strands: reasoning and problem solving; number and algebra; geometry and measures; and probability and statistics.

The reasoning and problem solving strand cuts across the other three strands. Reasoning, generalisation and problem solving should be an integral part of the teaching and learning of mathematics in all lessons.

The weightings of the content strands relative to each other are as follows:

Foundation	Number and algebra	Geometry and measures*	Probability and statistics
Grade 10	55%	30%	15%
Grade 11	55%	30%	15%
Grade 12	50%	25%	25%

* including trigonometry

The standards are numbered for easy reference. Those in shaded rectangles, e.g. 1.2, are the performance standards for all foundation students. The national tests for foundation mathematics will be based on these standards.

Many of the Grade 10 foundation standards have been introduced in earlier grades. Teachers should review and consolidate these standards, moving through them as quickly or as slowly as befits the students.

Mathematics standards

Grade 10 Foundation

Reasoning and problem solving

By the end of Grade 10, students solve routine and non-routine problems in a range of mathematical and other contexts, and use mathematics to model and predict the outcomes of real-world applications. They identify and use connections between mathematical topics. They break down complex problems into smaller tasks, and set up and perform appropriate manipulations and calculations. They develop and explain short chains of logical reasoning, using correct mathematical notation and terms. They generate simple mathematical proofs and identify exceptional cases. They aim to generalise. They approach problems systematically, knowing when it is important to enumerate all outcomes. They conjecture alternative possibilities with 'What if …?' and 'What if not …?' questions. They synthesise, present, interpret and criticise mathematical information, working to expected degrees of accuracy. They recognise when to use ICT and do so efficiently.

Students should:

1 Use mathematical reasoning to solve problems

1.1 Solve routine and non-routine problems in a range of mathematical and other contexts, including open-ended and closed problems.

1.2 Use mathematics to model and predict the outcomes of real-world applications; compare and contrast two or more given models of a particular situation.

1.3 Identify and use interconnections between mathematical topics.

1.4 Break down complex problems into smaller tasks.

1.5 Use a range of strategies to solve problems, including working the problem backwards and then redirecting the logic forwards; set up and solve relevant equations and perform appropriate calculations and manipulations; change the viewpoint or mathematical representation, and introduce numerical, algebraic, graphical, geometrical or statistical reasoning as necessary.

1.6 Develop short chains of logical reasoning, using correct mathematical notation and terms.

1.7 Explain their reasoning, both orally and in writing.

1.8 Generate simple mathematical proofs, and identify exceptional cases.

1.9 Learn to generalise and begin to understand the importance of generalisation in mathematics.

1.10 Approach a problem systematically, recognising when it is important to enumerate all outcomes.

1.11 Conjecture alternative possibilities with 'What if …?' and 'What if not …?' questions.

Key standards

Key performance standards are shown in shaded rectangles, e.g. 1.2.

Cross-references

Standards are referred to using the notation RP for reasoning and problem solving, NA for number and algebra, GM for geometry and measures, and PS for probability and statistics, e.g. standard NA 2.3.

Examples of problems

The examples of problems in italics are intended to clarify the standards, not to represent the full range of possible problems.

Reasoning and problem solving

Reasoning, generalisation and problem solving should be an integral part of the teaching and learning of mathematics in all lessons.

Proofs

Relate to the mathematics in the other strands.

1.12 Synthesise, present, discuss, interpret and criticise mathematical information presented in various mathematical forms.

1.13 Work to expected degrees of accuracy, and know when an exact solution is appropriate.

1.14 Recognise when to use ICT and when not to, and use it efficiently.

Number and algebra

By the end of Grade 10, students identify and use number sets and set notation. They calculate with any real numbers, including powers, roots, and numbers expressed in standard form. They use proportional reasoning to solve a range of problems involving scale, ratios and percentages. They are aware of the role of symbols in algebra. They generate and manipulate algebraic expressions, including algebraic fractions, equations and formulae. They sum arithmetic sequences and investigate the growth of simple patterns, generalising relationships to model the behaviour of the patterns. They find the solution of any linear equation, and a pair of simultaneous linear equations, and plot straight line and simple quadratic graphs. They use function notation. Through their study of linear and simple quadratic functions and their graphs, and the solution of the related equations, students begin to appreciate numerical and algebraic applications in the real world. They use realistic data and ICT to analyse problems.

Algebra

Students should learn that algebra enables generalisation and the establishment of relationships between quantities and/or concepts. They should understand the nature and place of algebraic reasoning and proof, and link it to geometric concepts wherever possible.

Students should:

2 Identify and use number sets

2.1 Identify the number sets:

$\mathbb{R}$ the set of all real numbers;

$\mathbb{Z}$ the set of all integers;

$\mathbb{Z}^+$ the set of all positive integers $\{1, 2, 3, 4, \ldots\}$;

$\mathbb{Z}^-$ the set of all negative integers;

$\mathbb{Q}$ the set of all rational numbers, i.e. all the different numbers that can be expressed in the form a/b, where a and b are integers with $b \neq 0$;

$\mathbb{N}$ the set of all non-negative integers, called the set of natural numbers $\{0, 1, 2, 3, 4, \ldots\}$.

Is $\mathbb{Z}$ a subset of $\mathbb{Q}$?

To what set does $\sqrt{2}$ belong? How do you know?

Natural numbers

In some texts, $\mathbb{N}$ is taken to be the same as $\mathbb{Z}^+$.

2.2 Know when a real number is irrational, i.e. when it is not a member of $\mathbb{Q}$.

2.3 Use and understand the following symbols associated with set theory: $\mathcal{E}$ for 'the universal set'; $\varnothing$ for 'the null set'; $\in$ for 'is a member of'; $\notin$ for 'is not a member of'; $\forall$ for 'for all'; use brace notation to denote a set.

$A = \{x: x \in \mathbb{R}$ and $1 \leq x < 10\}$ denotes 'the set A, whose members are all real numbers greater than or equal to 1 and less than 10'.

List the elements of each of the following sets:
A = {x: x is a colour of the Qatar flag};
B = {x: x is a state in the GCC};
C ={x: x is a member of the Arab League}.

Is the statement that $\sqrt{2}/_3 \in \mathbb{Q}$ *true or false?*

What is the solution set of the equation $x(x + 3) = x(x – 3) + 6x + 1$*?*
Explain your answer.

2.4 Understand the meaning of the *union of two sets A and B* and that this is denoted by $A \cup B$, and the meaning of the *intersection of two sets A and B*, denoted by $A \cap B$, and represent these sets in a Venn diagram; represent the *complement* of set A as A' and know that $A \cup A' = \mathcal{E}$.

Use a Venn diagram to decide whether $A \cup (B \cup C) = (A \cup B) \cup C$ *and* $A \cap (B \cap C) = (A \cap B) \cap C$.

Describe the set $\mathbb{Q} \cup \mathbb{Q}'$.

In a school of 650 students, everyone studies Arabic, English, mathematics and science. They all have to choose to study at least one of art, French, or history. 195 students choose only French. Three times as many students study French and history as study all three subjects, and five times as many study French and art as study all three subjects. 30 students study French and history, 65 do art and history, and 200 do art but not French or history. How many students study history but not art or French?

2.5 Know from definitions that every even number can be written in the form $2m$, where m is an integer, and that every odd number can be written in the form $2n + 1$, where n is an integer; understand and use the words *factor*, *multiple*, *divisor*, *prime number*, *prime factor*, *prime factor decomposition*, *least common multiple*, *highest common factor* and *lowest common denominator*.

Prove that the product of two odd numbers is an odd number.

What is the largest prime number you can think of? How do you know it is prime?

Is there a largest prime number? Justify your answer.

What is the highest common factor of a^3b^2c *and* c^3b^2a*?*

3 Use index notation and solve numerical problems

3.1 Understand exponents and *n*th roots, and apply the laws of indices to simplify expressions involving exponents; use the x^y key on a calculator.

Without using a calculator, evaluate $(5^3)^4 \div 5^{10}$.

Use a calculator to evaluate 7^9.

Simplify $8^{1/3} \times 2^{-1}$.

Laws of exponents

For $a > 0$:
$a^x \times a^y = a^{x+y}$
$a^x \div a^y = a^{x-y}$
$(a^x)^y = a^{xy}$
$(a^{1/n})^n = a$
$a^0 = 1$

3.2 Know that a root that is irrational is an example of a *surd*, as are expressions containing the addition or subtraction of an irrational root; perform exact calculations with surds.

Calculate $(\sqrt{2} – 1)(\sqrt{3} – \sqrt{2})$.

3.3 Use standard form in appropriate situations: for exact calculations, to estimate results of calculations and to make comparisons.

To four significant figures, the speed of light is 299 800 000 metres per second. Write this in standard form.

Standard form

Use examples drawn from science, geography or other real world applications.

A car has a mass of 1200 kilograms and a length of 4.5 metres. The Earth has a mass of 5.98×10^{24} kg and a radius of approximately 6400 kilometres. Estimate the ratio of the mass of the Earth to the mass of the car and the ratio of the radius of the Earth to the length of the car.

Sir Isaac Newton (1642–1727) was a mathematician, physicist and astronomer.

a. *In his work on the gravitational force between two bodies, Newton found that he needed to multiply their masses together.*

Work out the value of the mass of the Earth multiplied by the mass of the Moon. Give your answer in standard form.

Mass of Earth = 5.98×10^{24} kg
Mass of Moon = 7.35×10^{22} kg

b. *Newton also found that he needed to work out the square of the distance between the two bodies.*

Work out the square of the distance between the Earth and the Moon. Give your answer in standard form.

Distance between Earth and Moon = 3.89×10^{5} km

c. *Newton's formula to calculate the gravitational force (F) between two bodies is*

$$F = \frac{Gm_1m_2}{R^2}$$

where G is the gravitational constant, m_1 and m_2 are the masses of the two bodies, and R is the distance between them.

Work out the gravitational force (F) between the Sun and the Earth using this formula with the information given below. Give your answer in standard form.

$m_1m_2 = 1.19 \times 10^{55}$ kg^2
$R^2 = 2.25 \times 10^{16}$ km^2
$G = 6.67 \times 10^{-20}$

3.4 Calculate with any real numbers, including mental calculations in appropriate cases.

Calculate mentally the value of 999 × 33.

3.5 Add, subtract, multiply and divide any two fractions and understand how to use a unit fraction as a multiplicative inverse.

3.6 Understand the multiplicative nature of proportional reasoning; form, simplify and compare ratios, and apply these in a range of problems, including mixtures, map scales and enlargements in one, two or three dimensions.

A recipe for six people includes a quarter of a kilogram of figs. How many kilograms of figs would be needed if the recipe were made in the same proportion for eight people?

A map is drawn to scale 1 : 190 000. Two places A and B are 3 cm apart on the map. How far apart are A and B?

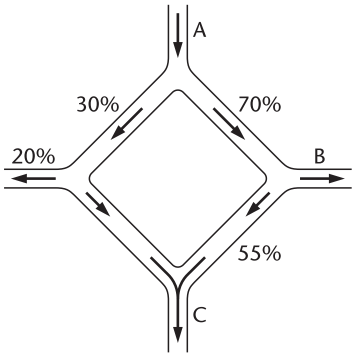
3.7 Perform percentage calculations, including finding a percentage of a percentage and inverse percentages.

After Haya's salary is increased by 15% and Abdullah's salary is decreased by 27%, Haya and Abdullah both end up with a salary of QR 36 000.
What were their original salaries?
What percentage of Abdullah's original salary was Haya's original salary?

The diagram shows water flowing through some pipes.
The water starts at A. At each junction the percentage of the inflowing water flowing out through the pipes is indicated.

What percentage of the original water flows out at B?

What percentage flows out at C?



Due to inflation, the price of a television in a store is increased by 15%. In the sales at the end of the year, the price is then reduced by 15%. Does the television revert to its original pre-inflation price? Or is it more, or less? Explain your reasoning.

4 Generate and manipulate algebraic expressions and formulae, and solve algebraic equations

4.1 Solve any linear equation with one unknown.

4.2 Generate sequences from term-to-term and position-to-term definitions; investigate the growth of simple patterns, generalising algebraic relationships to model the behaviour of the patterns.

Sequences

Link to geometric concepts where possible.

Include quadratic sequences and second order differences.

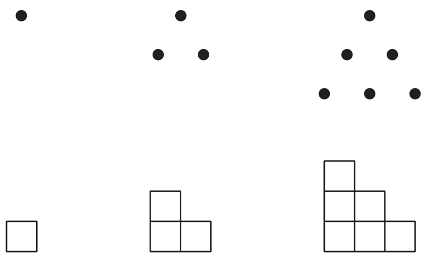
Each term of a sequence is 3 times the preceding term. The first term is 5. Set up a term-to-term definition for this sequence. Give an expression for the nth term in terms of n. Write down, but do not simplify, the 50th term.

The table shows the first six triangular numbers.

Position	1	2	3	4	5	6
Term	1	3	6	10	15	21

Investigate diagrammatic ways of representing triangular numbers.

The diagram shows some ways in which this might be done.



Set up a relationship to describe the nth term in terms of its position value n.
What is the 100th triangular number? What is the 1000th triangular number?

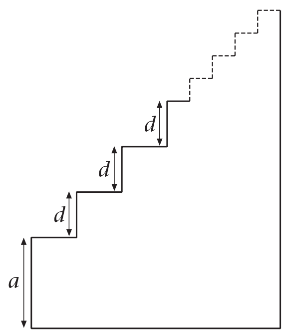
4.3 Sum arithmetic sequences, including the first *n* consecutive integers, and give a 'geometric proof' for the formulae for these sums.

ICT opportunity

Include the use of spreadsheets or graphics calculators to explore arithmetic sequences.

The diagram is a useful representation of an arithmetic series.

How could you use this diagram to find the sum of the arithmetic series?



Find the sum of the first n consecutive positive integers, and hence the sum of any set of n consecutive positive integers.

Find the sum of all numbers between 1 and 100 that are exactly divisible by 3.

4.4 Identify number patterns contained in Pascal's triangle.

Describe carefully in words how any entry in Pascal's triangle is related to entries in the row above. Set up an algebraic relationship to describe this. Where are the triangular numbers located in Pascal's triangle? What other patterns can you spot?

Look at the numbers in an early row of Pascal's triangle. Sum the squares of these numbers. In what row is the answer located? Identify where to find the sum of the squares of the numbers in any row of Pascal's triangle. Explain your reasoning.

4.5 Distinguish the different roles played by letter symbols in algebra, and understand that the transformation of algebraic objects generalises the well-defined rules of arithmetic. Recognise that letters are used to represent:

- the solution set of initially unknown numbers in *equations*;
- defined variables in *formulae*;
- generalised independent numbers in *identities*;
- new equations, expressions or functions in terms of known, or given, expressions or functions.

Is $(x + 4)^2 = x(x + 12) - 4(x - 4)$ *an equation or an identity? Explain your reasoning.*

Algebraic symbols

Include references and Internet research on the contributions to algebra of Arab scholars such as Al-Khwarizmi.

4.6 Use brackets and correct order of precedence of operations when performing numerical or algebraic calculations.

4.7 Multiply any combinations of monomial and binomial expressions, collecting and simplifying similar terms.

4.8 Simplify and combine numeric or algebraic fractions, including by cancelling common factors; rationalise a denominator of a fraction when the denominator contains simple combinations of surds.

Rationalise the expression $\frac{2}{1+\sqrt{3}}$.

Simplify the expression $\frac{a^3b^2 - a^2b^3}{a^2b^2}$.

4.9 Generate formulae from a physical context; rearrange formulae connecting two or more variables.

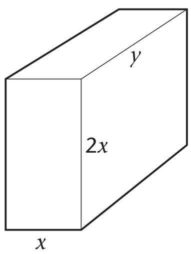
Formulae

Include examples drawn from science.

The three different edges of a solid cuboid have lengths x, 2x and y, as shown. All the lengths are measured in centimetres.

The total surface area of the cuboid is 800 cm^2*. Find a formula for y in terms of x.*

What is the total length of all the edges of the cuboid? Give the answer in terms of x.



Make b the subject of the formula $A = \frac{a+b}{2}$.

Make l the subject of the formula $T = 2\pi\sqrt{\frac{l}{g}}$.

Make x the subject of the formula $w = -z + \frac{x}{v}$.

5 Generate and solve problems with functions and graphs

5.1 Use function notation; investigate a range of mathematical and physical situations to develop the concepts of *function*, *domain* and *range*, recognising one-to-one and many-to-one mappings as functions and a one-to-many mapping as not a function.

If p is a person, state with reasons whether each of the following maps are functions:

a. *p maps to the place of birth of p;*
b. *p maps to brother of p;*
c. *p maps to nationality of p;*
d. *p maps to teacher of p;*
e. *p maps to mother of p.*

A firm rents out cars by the day or by the week. The daily charge rate is QR 170 with 150 km free and then QR 2 for every additional kilometre. The weekly charge is QR 1400 with no additional charges. A man needs to hire a car for five days. How many kilometres will he have to drive to make it worthwhile to hire the car for a week?

Look up any country in an atlas and pick six towns from it. Which of these maps represents a function and which does not: towns → country; country → towns? Justify your answer. What are the domain and range for the mapping that represents a function?

Functions

The notation $y = f(x)$ denotes that y is a function of x.

5.2 Understand and use the concept of related variables and, in special cases, set up appropriate functional relationships between them.

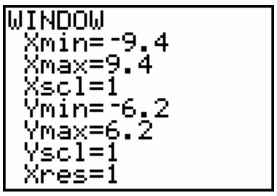
In an electric circuit, $V = IR$, where V is the voltage in volts, I is the current in amps and R is the resistance in ohms. The electrical power in watts is $P = VI$. Find a formula connecting the variables P, V and R.

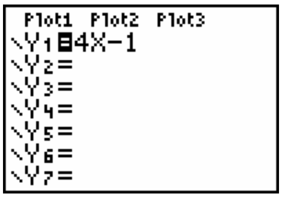
Functional relationships

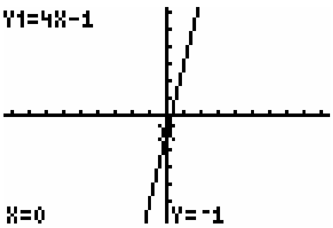
Include examples drawn from science.

5.3 Plot a graph to show the relationship between two variables given quantitative information between the variables in tabular or algebraic form.

5.4 Use a graphics calculator to plot a range of simple functional relationships, some continuous and others discontinuous, arising in familiar contexts.







Draw a graph showing the functional relationship between postage rate in Qatar and the weight of package to be posted.

ICT opportunity

Graphics calculators or graph plotters can be used to explore a range of functional relationships.

5.5 Recognise when a graph represents a functional relationship between two variables and when it does not.

Direct proportion

5.6 Translate the statement 'y is proportional to x' into the symbolism $y \propto x$ and into the equation $y = kx$, and know that the graph of this equation is a straight line through the origin and that the constant of proportionality, k, is the gradient of this line.

5.7 Know that if two coordinate variables are connected by a straight line graph that passes through the origin of coordinates, then each coordinate variable is proportional to the other; use relevant information to find k.

5.8 Identify common examples of two linear quantities varying in direct proportion to each other.

Straight lines and linear functions

5.9 Know that a straight line in the explicit form $y = mx + c$ represents a function; plot the graphs of such functions, relating the *gradient of the line* and *intercept on the x- or y-axis* to the coefficients m and c.

Are there straight lines that do not represent functions? Justify your answer.

5.10 Construct the Cartesian equation of a straight line from its graph alone, or from the knowledge of the coordinates of two points on the line, or from the coordinates of one point on the line and the gradient of the line.

What is the equation of the straight line through the points $(5, –2)$ *and* $(–4, 3)$*? What is the gradient of this line? Where does it cross the y-axis? Where does it cross the x-axis?*

A triangle has vertices at the points $(1, 1)$, $(5, –4)$ *and* $(–3, 2)$. *Find the equation of each of its sides.*

5.11 Know the condition for two straight lines to be parallel or perpendicular, including the special cases of one of the lines being parallel to either axis.

Give equations of lines parallel and perpendicular to the line $y = 5x – 3$.

5.12 Read off the coordinates of the point of intersection, given the graphs of two intersecting straight lines; use algebraic means to find exactly the coordinates of the point of intersection of two straight lines, given their equations.

Find the intersection point of the line $y = 4x + 2$ *with the line* $y = 9 – 3x$.

Discuss whether two lines have no intersection, a unique intersection point, or infinitely many intersection points.

Discuss whether non-parallel lines in two dimensions must intersect.
What happens in three dimensions?

5.13 Interpret the solution set of the simultaneous equations E_1 and E_2, where E_1 and E_2 are the equations of two straight lines.

Here are the equations of some straight lines:
$y = 2x – 7$; $y = 7 – 2x$; $y = 2x + 9$; $y = 14 – 4x$;
$y = –10$; $y = –10 + 2x$; $x = 1$; $y = –0.5x + 8$.

List all the pairs of lines that are: a. *parallel to each other;* b. *perpendicular to each other;* c. *different representations of the same line. From the list, find pairs of lines that intersect in a unique point and find the intersection point in each case.*

Quadratic functions

5.14 Recognise a simple second-order polynomial in one variable, $y = ax^2 + c$, as a quadratic function; plot graphs of such functions (recognising that these are all parabolas), and pick out the intercepts with the coordinate axes, the axis of symmetry and the coordinates of the maximum or minimum point.

5.15 Model a range of situations with quadratic functions of the form $f(x) = ax^2 + c$.

Direct proportion

Use real examples, e.g.

- currency conversions;
- for a given resistance, the voltage in an electric circuit is directly proportional to the current;
- the pressure of gas in a constant volume is directly proportional to its temperature.

Gradients

Include the terms *slope* and *rate of change*.

ICT opportunity

Include the use of graphics calculators or graph plotters.

Geometry and measures

By the end of Grade 10, students use their knowledge of geometry, Pythagoras' theorem and the trigonometry of right-angled triangles to solve practical and theoretical problems relating to shape and space. They understand congruence and similarity. They carry out straight edge and compass constructions and determine the locus of an object moving according to a rule. They use a range of SI units and measures, including bearings and compound measures. They use formulae to calculate: the circumference and area of a circle; the perimeter and area of any triangle, or trapezium, parallelogram or quadrilateral with perpendicular diagonals: the surface area and volume of a right prism, cylinder, square-based pyramid and cone: and the volume of a sphere. They use ICT to explore pattern, similarity, congruence and constructions.

Geometry and measures

Students should develop an appreciation of the importance and range of geometrical applications in the real world, and the aesthetic qualities of geometric models. They should understand the nature and place of geometric reasoning and proof, and how geometry may be related to algebraic concepts, and vice versa.

Students should:

6 Develop geometrical reasoning and proof, and solve geometric problems

Congruence and similarity: properties of angles, straight lines and triangles

6.1 Use knowledge of angles at a point, angles on a straight line, and alternate and corresponding angles between parallel lines and a transversal line to present formal arguments to establish the congruency of two triangles.

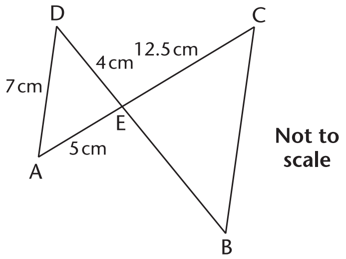
Prove that each of the angles in an equilateral triangle is 60°.

6.2 Establish the congruency of two triangles to generate further knowledge and theorems about triangles, including proving that the base angles of an isosceles triangle are equal and that the line joining the mid-points of two sides of a triangle is parallel to the remaining side.

6.3 Understand similarity of two triangles and other rectilinear shapes, knowing that similarity preserves shape and angles, but not size; make inferences about the lengths of sides and about the areas of similar figures; prove that if two triangles are similar, then the ratio of the areas of the two triangles is the square of the ratio of any pair of corresponding sides of the two triangles chosen in the same order; in three dimensions, calculate the ratio of the volume of a scale model to the volume of the actual object.

A goldsmith has a block of gold in the shape of a cube. He wants to make another gold cube that has exactly twice the volume of the first cube. What scale factor must he use?

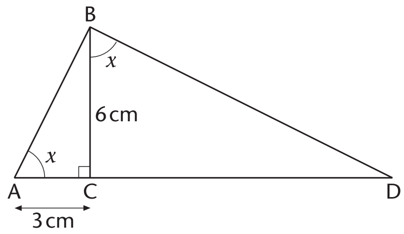
The diagram shows two triangles ADE and BCE.
Side AD is parallel to side BC.
Explain why the two triangles are similar to each other.
Calculate the missing lengths for triangle BCE.



Use of ICT

Geometry is enhanced with use of a dynamic geometry system, or DGS, which provides an interactive focus to investigate and conjecture results which could then be proved as theorems.

Calculate the length of CD in the diagram.



Two similar shaped gas-filled balloons are made of a special material. The area of material used in one balloon is 100 cm^2. The material for the other balloon has an area of 225 cm^2. Calculate the ratio of the volume of the larger balloon to the volume of the smaller balloon. Give this ratio in its simplest form.

A scale model of a dhow has a volume of 300 cm^3. The length of the actual dhow is 100 times longer than the length of the model. What is the volume of the dhow? Give the answer in appropriate units.

6.4 Calculate the interior and exterior angles of regular polygons; name polygons with up to ten sides.

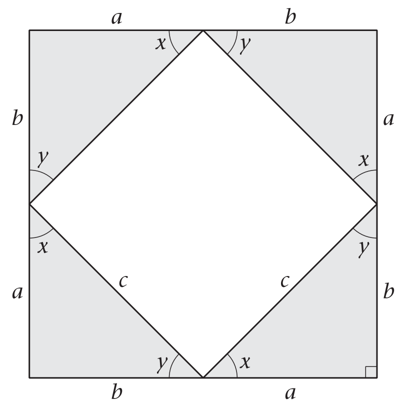
Trigonometry, Pythagoras' theorem and the solution of triangles

6.5 Know the standard trigonometric ratios, and their standard abbreviations, for sine of θ, cosine of θ and tangent of θ, given an angle θ in a right-angled triangle; use these ratios to find the angles of a right-angled triangle given two sides, or to find the remaining sides given one side and one angle.

Show that $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$.

6.6 Know at least two different proofs of Pythagoras' theorem.

Explain how you could use the diagram to prove Pythagoras' theorem.



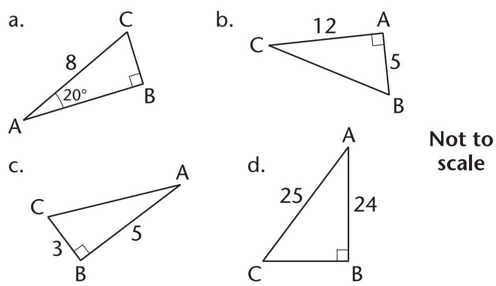
Pythagoras' theorem

There are many websites devoted to proofs of this fundamental theorem in geometry.

6.7 Use Pythagoras' theorem to find the distance between two points and to solve right-angled triangles; set up the Cartesian equation of a circle of radius *r*, centred at the origin of an *xy*-coordinate system.

Show that a triangle with sides of length $m^2 - n^2$, $2mn$ *and* $m^2 + n^2$ *respectively is always right-angled. Find some right-angled triangles using this result.*

Solve the triangles shown. Give all the angles and all the sides.



Each side of a cube is 5 cm. Calculate the length of a diagonal of the cube from one vertex on the 'base' to the opposite vertex on the 'top face'. What is the angle between this diagonal and the base?

Constructions

6.8 Perform and justify straight edge and compass constructions, including those to bisect a line, to construct an equilateral triangle with a side of given length, to drop a perpendicular from a point to a line, and to bisect an angle.

Construct a square. You may only use a straight edge, a pencil and a pair of compasses.

Use the construction to bisect an angle several times over to construct an angle of 22.5°.

Explain why the construction to bisect an angle works.

Loci

6.9 Determine the locus of an object moving according to a rule, including those arising in simple physical situations.

A goat is on a rope attached at one corner of a rectangular enclosure. The enclosure measures 10 m by 4 m. The rope is 6 m long. Draw a scale drawing of the enclosure and shade in the locus in which the goat can move.

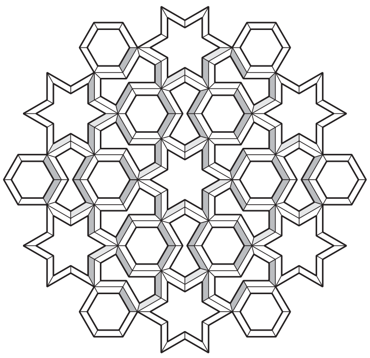
Find the locus of all points 3 cm from a circle of radius 5 cm. Discuss how the locus is changed if three dimensions are allowed.

Transformations

6.10 Investigate Islamic patterns and describe their features.

The picture shows a pattern from a mosque in Isfahan, in Iran.

Use this and other Islamic patterns to discuss key features of the pattern (its construction, reflection symmetries, translations, and so on).



Transformations

Transformations are best developed through use of DGS.

Use of ICT

6.11 Use ICT to explore geometrical relationships.

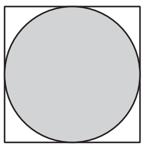
7 Use a range of measures and compound measures to solve problems

7.1 Use formulae to calculate: the circumference and area of a circle; the perimeter and area of any triangle, or trapezium, parallelogram or quadrilateral with perpendicular diagonals; the surface area and volume of a right prism, cylinder, square-based pyramid and cone; and the volume of a sphere.

The diagram shows a circle drawn inside a square. The radius of the circle is 5 cm.

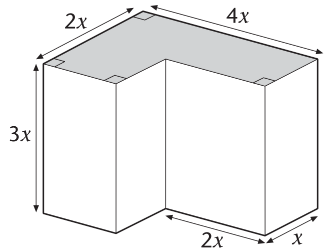
Find the area of the square.

Calculate the exact ratio of the area of the square to the area of the circular region.



The solid is a prism, with dimensions as shown. The cross-section is shaded.

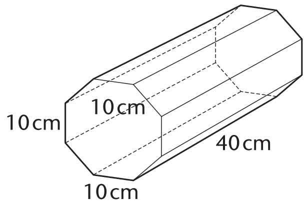
Calculate the volume of this prism.



The diagram shows the frame of a kite. The frame is made of sticks and forms a right prism with octagonal ends.

The lengths of some sticks are marked on the diagram.

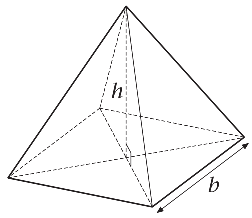
What is the total length of all the sticks?



The volume of a pyramid is $\frac{1}{3}$(base area × perpendicular height).

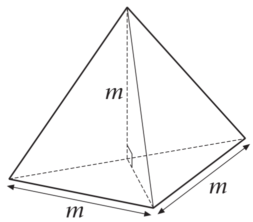
Calculate the volume of a pyramid with a square base of side 5 cm and perpendicular height 6 cm.

Another pyramid has a square base with side 4 cm and a volume of 48 cm^3. What is its perpendicular height?



The diagram shows a pyramid with a triangular base that is an isosceles right-angled triangle.

Write down a formula for the volume of this pyramid.



7.2 Use bearings.

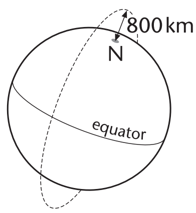
An oil tanker sails 350 km from Doha towards Dubai on a bearing of 090° and then from Dubai towards Al Kuwayt on a bearing 310°.
Al Kuwayt is about 600 km from Doha.
Approximately, how far is it from Dubai to Al Kuwayt?

7.3 Work with SI units and compound measures: rates such as cost per litre, kilometres per litre, litres per kilometre; and average speed and density, including population density (number of people per unit area).

A satellite passes over both the north and south poles, and it travels 800 km above the surface of the Earth. The satellite takes 100 minutes to complete one orbit.

Assume the Earth is a sphere and that the diameter of the Earth is 12 800 km.

Calculate the speed of the satellite, in kilometres per hour.



Compound measures

Use appropriate SI units and dimensions. Stress how units are calculated in compound measures.

Probability and statistics

By the end of Grade 10, students know that statistical data are collected from observation or measurement on samples taken from a larger population, and that, by analysing the sample data, inferences can be made about the population as a whole. They distinguish between qualitative (or categorical) data and quantitative data, and between discrete and continuous data. They plan simple surveys, design simple questionnaires and plot histograms in which the height of each bar is proportional to the frequency of that class. They calculate means and medians, and understand mode and modal class. They plot and interpret simple scatter diagrams between two random variables, and draw a line of best fit where there appears to be correlation. They use relevant statistical functions on a calculator and ICT applications to present statistical tables and graphs.

Probability and statistics

Students should know that statistics is the branch of mathematics used to predict the outcomes of large numbers of events when these outcomes are uncertain, and that probability lies at the heart of statistics. They should be aware of the uses of statistics in society and recognise when statistics are used sensibly and when they are misused or likely to be misunderstood.

Students should:

8 Collect, process, represent, analyse and interpret data and reach conclusions

Introductory statistical techniques

Statistical techniques

Include primary data collected in other subjects, such as science and the social sciences.

Include secondary data downloaded from the Internet.

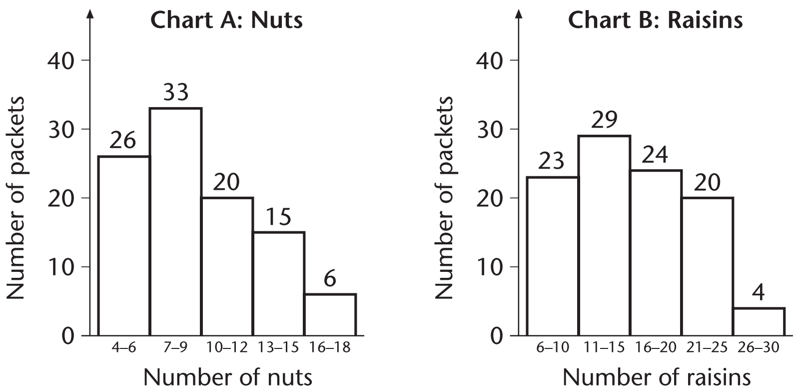
8.1 Know that different types of data can be collected from samples – qualitative or categorical data (e.g. eye colour, male, female) and quantitative data (e.g. age, height, lifespan, mortality rates) – and that quantitative data may be discrete (e.g. number of defective items in a production process) or continuous (e.g. weight); understand the concept of a random variable.

8.2 Plan simple surveys and design questionnaires to collect meaningful primary data from samples in order to test simple hypotheses about, or estimate, characteristics of the population as a whole; formulate problems using secondary data from published sources, including the Internet.

8.3 Plot simple histograms in which the height of the bars is proportional to the frequency of each class interval and use related vocabulary, including *frequency*, *range* and *mode*, *modal class* and *modal frequency*.

8.4 Calculate measures of central tendency such as the arithmetic mean and the median.

A company makes breakfast cereal containing nuts and raisins.
They counted the number of nuts and raisins in 100 small packets.



a. *Calculate an estimate of the mean number of nuts in a packet.*
You may complete the table below to help you with the calculation.

Number of nuts	Mid-point of bar (x)	Number of packets (f)	fx
4–6	5	26	130
7–9	8	33	
10–12	11	20	
13–15	14	15	
16–18	17	6	
		100	

b. *Calculate an estimate of the number of packets that contain 24 or more raisins.*

8.5 Make simple inferences and draw conclusions from the formulation of a problem to the analysis of data in a range of simple situations.

Compare the television viewing habits of students in different grades at school.

9 Simple correlation

9.1 Draw scatter diagrams between two random variables associated with some common context; identify through elementary qualitative discussion positive and negative correlation; where there appears to be correlation, draw a line of best fit, judging by eye the line about which the data points are most evenly distributed.

ICT opportunity

Use a graphics calculator to draw a scatter plot, or a spreadsheet application with graphs.

Compare the examination marks for all students in a class for:

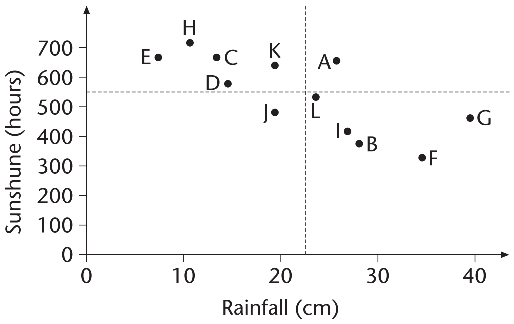
a. *mathematics and science;*

b. *Arabic and English;*

c. *mathematics and art.*

Discuss whether there appears to be correlation or not.

The scatter diagram shows the total amounts of sunshine and rainfall for 12 seaside towns in the UK during one summer. Each town has been given a letter. The dashed lines go though the mean amounts of sunshine and rainfall.



Which town's rainfall was closest to the mean?

Draw a line of best fit on the scatter diagram. Use your line to find an estimate of the hours of sunshine for a seaside town that had 30 cm of rain.

10 Use of ICT

10.1 Use a calculator with statistical functions to aid the analysis of large data sets, and ICT applications to present statistical tables and graphs.