

Summary of students' performance by the end of Grade 11

Reasoning and problem solving

Students solve routine and non-routine problems in a range of mathematical and other contexts. They use mathematics to model and predict outcomes of real-world applications, and compare and contrast two or more given models of a particular situation. They break down complex problems into smaller tasks, and set up and perform appropriate calculations and manipulations. They identify and use connections between mathematical topics. They develop and explain chains of logical reasoning, using correct mathematical notation and terms. They generate mathematical proofs and generalise when possible. They approach problems systematically, knowing when to enumerate all outcomes. They conjecture alternative possibilities with 'What if ...?' and 'What if not ...?' questions. They synthesise, present, interpret and criticise mathematical information, working to expected degrees of accuracy. They recognise when to use ICT and do so efficiently.

Number and algebra

Students use proportional reasoning and harder percentage calculations to solve problems, including inverse proportion problems. They use sigma notation and generate and sum simple recursive sequences, including arithmetic and geometric series. They find combinations and permutations, and use the binomial theorem expansion of $(1 + x)^n$, where n is a positive integer. They form and manipulate algebraic expressions and formulae, simplify and combine algebraic fractions, rearrange harder formulae and generate further formulae from physical contexts. They find approximate solutions of quadratic equations, and a pair of simultaneous equations, one linear and one quadratic. They use physical contexts to plot and interpret the graphs of linear, quadratic, cubic, reciprocal, exponential and logarithm functions, the sine and cosine functions, and the modulus function. They recognise symmetry properties of functions, when they are even or odd, and when they are increasing, decreasing or stationary. They solve problems using inverse and composite functions. They apply combinations of transformations to the graph of the function $y = f(x)$. They understand the concept of a limit and find derivatives of functions. Through their study of functions and their graphs, and the solution of associated equations, students appreciate a range of numerical and algebraic applications in the real world. They continue to use realistic data and ICT to analyse problems.

Geometry and measures

Students use geometry and trigonometry to solve practical and theoretical problems. They know and use the sine and cosine rules, and calculate the area of a triangle using $\frac{1}{2} ab \sin C$. They use Pythagoras' theorem to show that $\sin^2 \theta + \cos^2 \theta = 1$ for any angle θ . They find the points of intersection of a straight line with a circle. They plot the graphs of circular functions and solve simple problems modelled by these functions. They prove standard circle theorems. They draw and use plans and elevations. They translate, reflect, rotate and enlarge two-dimensional geometric objects. They begin to use vectors to solve physical problems. They solve a range of problems involving compound measures, using appropriate units and dimensions. They explore

geometry using ICT.

Probability and statistics

Students use a simple mathematical model to calculate, for a particular set of events, the theoretical probability of obtaining a particular outcome for a random variable associated with those events. They calculate probabilities of single and combined events, and understand risk as the probability of the occurrence of an adverse event. They use tree diagrams to represent and calculate the probabilities of compound events when the events are independent and when one event is conditional on another. They use simple simulations and consider trends over time using a moving average. They analyse results to draw conclusions and use a range of graphs, charts and tables to present their findings. They use relevant statistical functions on a calculator and ICT applications to present statistical tables and graphs.

Content and assessment weightings for Grade 11

The advanced mathematics standards have four strands: reasoning and problem solving; number, algebra and calculus; geometry and measures; and probability and statistics. Calculus is introduced in Grade 12 in the advanced standards.

The reasoning and problem solving strand cuts across the other three strands. Reasoning, generalisation and problem solving should be an integral part of the teaching and learning of mathematics in all lessons.

The weightings of the content strands relative to each other are as follows:

Advanced	Number, algebra and calculus	Geometry, measures and trigonometry	Probability and statistics
Grade 10	55%	30%	15%
Grade 11	55%	30%	15%
Grade 12 (quantitative)	40%	–	60%
Grade 12 (for science)	75%	25%	–

The standards are numbered for easy reference. Those in shaded rectangles, e.g. 1.2, are the performance standards for all advanced students. The national tests for advanced mathematics will be based on these standards.

Grade 11 teachers should review and consolidate Grade 10 standards where necessary.

Reasoning and problem solving

Reasoning and problem solving

By the end of Grade 11, students solve routine and non-routine problems in a range of mathematical and other contexts. They use mathematics to model and predict outcomes of real-world applications, and compare and contrast two or more given models of a particular situation. They break down complex problems into smaller tasks, and set up and perform appropriate calculations and manipulations. They identify and use connections between mathematical topics. They develop and explain chains of logical reasoning, using correct mathematical notation and terms. They generate mathematical proofs and generalise when possible. They approach problems systematically, knowing when to enumerate all outcomes. They conjecture alternative possibilities with ‘What if ...?’ and ‘What if not ...?’ questions. They synthesise, present, interpret and criticise mathematical information, working to expected degrees of accuracy. They recognise when to use ICT and do so efficiently.

Students should:

- 1 Use mathematical reasoning to solve problems**
 - 1.1** Solve routine and non-routine problems in a range of mathematical and other contexts, including open-ended and closed problems.
 - 1.2** Use mathematics to model and predict the outcomes of real-world applications; compare and contrast two or more given models of a particular situation.
 - 1.3** Identify and use interconnections between mathematical topics.
 - 1.4** Break down complex problems into smaller tasks.
 - 1.5** Use a range of strategies to solve problems, including working the problem backwards and then redirecting the logic forwards; set up and solve relevant equations and perform appropriate calculations and manipulations; change the viewpoint or mathematical representation, and introduce numerical, algebraic, graphical, geometrical or statistical reasoning as necessary.
 - 1.6** Develop chains of logical reasoning, using correct mathematical notation and terms.
 - 1.7** Explain their reasoning, both orally and in writing.
 - 1.8** Generate mathematical proofs, and identify exceptional cases.
 - 1.9** Generalise whenever possible.
 - 1.10** Approach a problem systematically, recognising when it is important to enumerate all outcomes.
 - 1.11** Conjecture alternative possibilities with ‘What if ...?’ and ‘What if not ...?’ questions.

Key standards

Key performance standards are shown in shaded rectangles, e.g. **1.2**.

Cross-references

Standards are referred to using the notation RP for reasoning and problem solving, NA for number and algebra, GM for geometry and measures and PS for probability and statistics. e.g. standard NA 2.3

Examples of problems

Examples of problems in italics are intended to clarify the standards, not to represent the full range of possible problems.

Reasoning and problem solving

Reasoning, generalisation and problem solving should be an integral part of the teaching and learning of mathematics in all lessons.

Proofs

Relate to the mathematics in the other strands.

Generalisation

Students should appreciate that generalisation is important in mathematics.

- 1.12** Synthesise, present, discuss, interpret and criticise mathematical information presented in various mathematical forms.
- 1.13** Work to expected degrees of accuracy, and know when an exact solution is appropriate.
- 1.14** Recognise when to use ICT and when not to, and use it efficiently.

Number and algebra

By the end of Grade 11, students use proportional reasoning and harder percentage calculations to solve problems, including inverse proportion problems. They use sigma notation and generate and sum simple recursive sequences, including arithmetic and geometric series. They find combinations and permutations, and use the binomial theorem expansion of $(1 + x)^n$, where n is a positive integer. They form and manipulate algebraic expressions and formulae, simplify and combine algebraic fractions, rearrange harder formulae and generate further formulae from physical contexts. They find approximate solutions of quadratic equations, and a pair of simultaneous equations, one linear and one quadratic. They use physical contexts to plot and interpret the graphs of linear, quadratic, cubic, reciprocal, exponential and logarithm functions, the sine and cosine functions, and the modulus function. They recognise symmetry properties of functions, when they are even or odd, and when they are increasing, decreasing or stationary. They solve problems using inverse and composite functions. They apply combinations of transformations to the graph of the function $y = f(x)$. They understand the concept of a limit and find derivatives of functions. Through their study of functions and their graphs, and the solution of associated equations, students appreciate a range of numerical and algebraic applications in the real world. They continue to use realistic data and ICT to analyse problems.

Algebra

Students should learn that algebra enables generalisation and the establishment of relationships between quantities and/or concepts. They should understand the nature and place of algebraic reasoning and proof, and how algebra may be related to geometric concepts, and vice versa.

Students should:

2 Identify and use number sets

- 2.1 Make appropriate use of knowledge of number sets.

3 Use index notation and solve numerical problems

- 3.1 Develop further confidence in using the rules for indices.

The atmospheric pressure on Mercury is $2 \times 10^{-8} \text{ N/m}^2$. The atmospheric pressure on Earth is 5.05×10^{12} times as great as the atmospheric pressure on Mercury. Calculate the atmospheric pressure on Earth.

Farida is making a scale model of the Earth and the Moon for a museum. She has found out the diameters of the Earth and the Moon, and the distance between them in metres.

Diameter of the Earth	$1.28 \times 10^7 \text{ m}$
Diameter of the Moon	$3.48 \times 10^6 \text{ m}$
Distance between Earth and Moon	$3.89 \times 10^8 \text{ m}$

- a. How many times bigger is the diameter of the Earth than the diameter of the Moon?
- b. In Farida's scale model the diameter of the Earth is 50 cm. What should be the distance between the Earth and the Moon in Farida's model?

Laws of exponents

For $a > 0$:

$$a^x \times a^y = a^{x+y}$$

$$a^x \div a^y = a^{x-y}$$

$$(a^x)^y = a^{xy}$$

$$(a^{1/n})^n = a$$

$$a^0 = 1$$

4 Work with sequences, series, recurrence relations and arrangements

- 4.1** Generate sequences from term-to-term definitions and from position-to-term definitions, including recursive sequences, to model the behaviour of the real-world situations, for example population growth.

In a TV quiz show a contestant can triple her winnings if she survives from one round to the next. Write down a formula for her prize P_{n+1} in the $(n + 1)$ th round in terms of her prize P_n in the n th round. Write an alternative formula for P_{n+1} in terms of n . The prize for winning in the first round is QR 1000. What is the minimum number of rounds that will have to be contested to win at least QR 700 000?

A sequence is defined by $u_{n+2} = u_{n+1} - u_n$ with $u_1 = 5$ and $u_2 = -4$. Write down the first eight terms of the sequence.

A sequence is defined by $u_n = n(n - 1) + 41$. Write down the first twelve terms of this sequence. What do you notice about these terms? Form a conjecture about this sequence and carry out further tests to see if your conjecture is correct.

In a certain country, there is a net increase in population from one year to the next of 5 per cent. Set up a recurrence relation to describe the population in year $n + 1$ in terms of the population in year n . Find the population in year $n + 4$ compared to the population in year n . Use your formula to find the number of years it takes to double the population from year n .

A woman buys a car and pays in monthly instalments. The car costs QR 60 500 and interest is charged on any outstanding debt at a monthly rate of $r\%$. The woman pays back a fixed amount each month of QR M . Set up a recurrence relation connecting the amount owed, A_{n+1} , after $n + 1$ months in terms of the amount owed, A_n , at the end of the n th month. How many months will it take to repay the debt if $M = \text{QR } 1200$ and $r = 1.2\%$? How much will the woman have then paid for the car? Investigate repayments for different values of M and r .

- 4.2** Understand and use sigma notation for summing the terms of a sequence.

Find $\sum_1^{10} (r + 1)$.

Find $\sum r^2$ for integer values of r from 1 to 10.

Write out in full $\sum_1^4 \frac{(-1)^r}{r(r+1)}$ and use partial fractions to evaluate this sum.

Rewrite these sums using sigma notation.

$$7^2 + 8^2 + 9^2 + 10^2$$

$$\frac{1}{121} - \frac{1}{144} + \frac{1}{169} - \frac{1}{196} + \frac{1}{225}$$

- 4.3** Recognise an arithmetic progression (AP); sum an arithmetic series and know the formula for the r th term of the series in terms of the first term and the common difference between terms.

The fourth term of an AP is 25. The sum of the first six terms of the AP is 120. Find the eleventh term.

- 4.4** Recognise a geometric progression (GP); generate term-to-term and position-to-term definitions for the terms of a GP in terms of the common ratio between terms; sum a finite geometric series.

At the end of every year a car loses 30 per cent of its value at the start of the year. Construct a formula, in terms of the original purchase price, to give the value of the car n years after purchase. After how many years will the car first be worth less than 90 per cent of its original value?

Recurrence relations

Term-to-term definitions of sequences are also known as *recurrence relations*.

ICT opportunity

Use spreadsheets in examples like these.

Sigma notation

The start and end values of the positions of the terms to be summed are commonly indicated below and above the sigma sign respectively.

- 4.5** Sum to infinity a convergent geometric series and know the condition on the common ratio for an infinite geometric series to be convergent.

The first term of an infinite geometric series is 1 and the common ratio is $\frac{1}{2}$. Write down the first six terms and find the sum of the series.

In another infinite geometric series, the first term is 1 and the common ratio is $-\frac{1}{2}$. Write down the first six terms and find the sum of the series.

- 4.6** Understand and use formulae for the sum of:
the squares of the first n positive integers;
the cubes of the first n positive integers.

- 4.7** Understand and use factorial notation and know that $0! = 1$; know the binomial theorem expansion of $(1 + x)^n$ for positive integer n and that the term in x^r has coefficient ${}^n C_r$, where ${}^n C_r = \frac{n!}{(n-r)!r!}$; know how to use Pascal's triangle to find ${}^n C_r$; find permutations and combinations.

Show that ${}^n C_r = \frac{n(n-1)(n-2)\dots(n-r+1)}{r!}$.

Explain why $(a + b)^n = \sum_{r=0}^n {}^n C_r a^{n-r} b^r$ for any positive integer n .

Use the binomial theorem to expand $(x - 2y)^4$.

5 Generate and solve problems with functions and graphs

- 5.1** Understand the symmetry properties of functions, and when a function is even or odd; sketch and describe the features of polynomial functions up to order 3.

Use symmetry properties to help sketch graphs of polynomial functions up to order 3. What other considerations are helpful in sketching curves?

State whether the following functions are even, or odd, or neither even nor odd.

$$f(x) = x^3 + 2x^2 - 5$$

$$g(x) = \frac{x^3}{1-x^2}$$

Quadratic functions

- 5.2** Model a range of situations with appropriate quadratic functions.

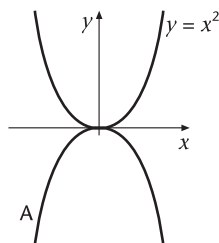
A fountain at ground level sprays out jets of water. Each jet is a parabola. The jet that sprays the farthest has equation $y = -x^2 + 8x - 15$. Factorise this expression.

Hence find a. where the fountain jet is positioned in this xy -coordinate system and b. how far from the fountain jet the water hits the ground.

Calculate the greatest height that the water reaches.

Curve A is the reflection in the x -axis of $y = x^2$.

What is the equation of curve A?



Binomial theorem

The theorem has important applications in probability theory.

An alternative notation

The notation $\binom{n}{r}$ is often

used for the expression $\frac{n(n-1)(n-2)\dots(n-r+1)}{r!}$

when n is not a positive integer.

Functions

Include use of the notation $f : x \mapsto f(x)$, as well as

$y = f(x)$ or $f(x) = \dots$

Modelling with functions and their inverses

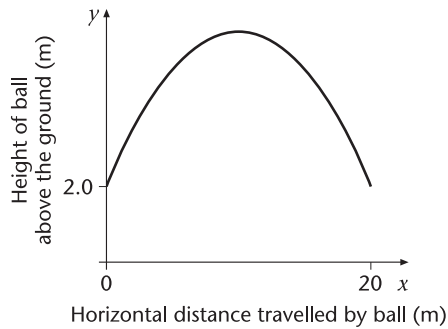
This aspect of mathematics adds realism, and shows the importance of the subject through application.

ICT opportunity

Where possible, use real data and analyse the data using ICT.

Huda throws a ball to Mariam who is standing 20 m away.
The ball is thrown and caught at a height of 2.0 m above the ground.

The ball follows the curve with equation $y = 6 + c(10 - x)^2$, where c is a constant. Calculate the value of c by substituting $x = 0, y = 2$ into the equation.



An n -sided polygon has $\frac{1}{2}n(n - 3)$ diagonals. How many diagonals has an octagon?
A polygon has 104 diagonals. How many sides does it have?

- 5.3** Find the axis of symmetry of the graph of a quadratic function, and the coordinates of its turning point by algebraic manipulation; understand the effect of varying the coefficients a, b and c in the expression $ax^2 + bx + c$.

$y = (x - 3)^2 + 5$ is a quadratic function of x . What is the minimum value of this function and for what value of x does it occur? What is the maximum range of the function? Give the equation of the axis of symmetry of the function. Write an alternative form for the equation defining the function. Sketch the graph of this function.

- 5.4** Given a quadratic equation of the form $ax^2 + bx + c = 0$, know that:

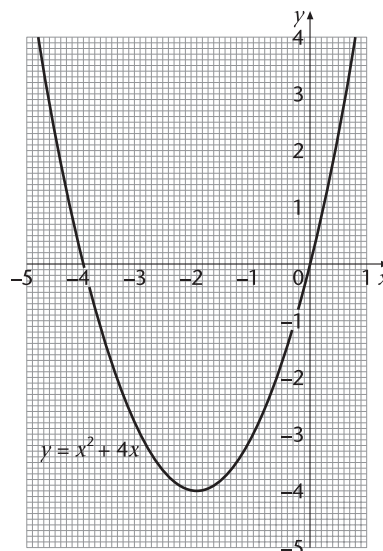
- the discriminant $\Delta = b^2 - 4ac$ must be non-negative for the exact solution set in \mathbb{R} to exist;
- there are two distinct roots if Δ is positive and one repeated root if Δ is zero.

- 5.5** Find approximate solutions of the quadratic equation $ax^2 + bx + c = 0$ by reading from the graph of $y = ax^2 + bx + c$ the x -coordinate(s) of the intersection point(s) of the graph of this function and the x -axis.

ICT opportunity
Include the use of a graphics calculator.

The graph shows the curve $y = x^2 + 4x$.

- Solve the equation $x^2 + 4x - 2 = 0$ using the graph. Give your answers to two decimal places.
- The equation $x^2 + 4x + 5 = 0$ cannot be solved using the graph. Why not?



- 5.6** Solve equations and inequalities using algebra or a combination of algebra and graphical representation.

Find the solution set of $(x - 1)^2 \geq 9$.

Solve the inequality $|2x - 3| \leq x + 3$.

- 5.7** Use the graph of the function $f(x) = ax^2 + bx + c$ to determine regions where $ax^2 + bx + c$ is greater than or less than zero.

Solve the inequality $x^2 + x - 6 > 0$.

- 5.8** Find exactly by analytical methods and approximately by graphical methods, the solution set of two simultaneous equations L_1 and Q_1 , where L_1 represents a linear relation for y in terms of x , and Q_1 a quadratic function of y in terms of x .

How can you determine how many points of intersection a given straight line $y = mx + c$ has with a given quadratic curve $y = ax^2 + bx + c$?

Find the intersection points of the straight line $4y = 3x$ and the circle $(x - 4)^2 + y^2 = 9$.

- 5.9** Solve physical problems modelled simultaneously by two such functions.

Inverse proportion and the reciprocal function

- 5.10** Understand the statement y is *inversely proportional to x* and set up the corresponding equation $y = k/x$; know some characteristics, including that $x \neq 0$ and that $x = 0$ is an *asymptote* to the curve, as is $y = 0$; study examples of inverse proportionality.

ICT opportunity

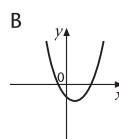
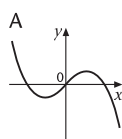
Include the use of a graphics calculator.

ICT opportunity

Include the use of a graphics calculator.

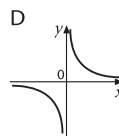
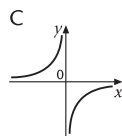
Look at these graphs.

- a. One of the graphs shows the equation $y = kx - x^2$ (k is a constant).

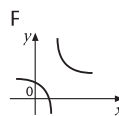
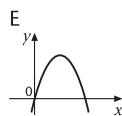


Which graph is it?

- b. One of the graphs shows the equation $y = k/x$, where k is a positive constant.



Which graph is it?



Explain why the function $y = k/x$ cannot be defined on the domain set \mathbb{R} . What is the largest domain the function can be defined on? Sketch the graph of the function for this domain. Does the function have a greatest or least value? Is there anywhere where the function increases?

Three people working flat out complete a job in sixteen hours. How many hours would it take eight people to do the same job? Explain any assumptions you have made.

The average speed for a fixed distance journey is inversely proportional to the time taken to complete the journey. A family travels in Europe by car. They travel exactly half their journey in 2 hours, then stop for lunch for 1 hour, and then take 3 hours over the second half of the journey. How were the average speeds related on each part of the journey? If the average speed for the first half of the journey was 72 kilometres per hour, what was the average speed for the whole journey?

- 5.11** Use a graphics calculator, including use of the trace function, to show approximate solutions to physical problems requiring the location and physical interpretation of the intersection points of two or more graphs.

- 5.12** Use physical contexts to plot and interpret graphs of functions, recognising when the functions are increasing, decreasing or stationary, including:

- linear, quadratic and cubic functions;
- the reciprocal function $y = k/x$ ($x \neq 0$);
- the sine and cosine functions;
- the modulus function and a range of simple non-standard functions.

Which grows faster for $x \geq 0$: the power function $y = x^3$ or the exponential function $y = e^x$? Justify your answer.

A big wheel makes one complete revolution every 90 seconds. The wheel has a diameter of 20 metres. The bottom of the wheel is 2 metres above the ground. Two people get on the wheel and sit in a seat, and then the wheel starts to rotate. T seconds later their height above the ground is given by $h = 2 + 8 \sin 4T^\circ$. Explain why this is an appropriate formula to use. At what two consecutive times are they 12 m above the ground?

A ship can only enter a harbour when the tide is in; it must have a minimum depth of water of 8 metres. The tide follows a daily sinusoidal variation given by the formula $d = 5 \sin 15t^\circ + 8$, where t is the time in hours from midnight onwards, measured on the 24-hour clock. At how many times in a day will the depth of water in the harbour be exactly 8 m? For how many hours a day can the ship enter the harbour? Sketch how the level of the tide varies with the time of the day.

Find physical examples that are modelled by circular functions.

- 5.13** Find, graph and use the inverse function of those functions given by a one-to-one mapping or restricted to such mappings; know that the graph of the inverse function may be found by reflecting the graph of the function in the line $y = x$; solve a range of problems using inverse functions.

The cost of production of q silver bracelets is $C = 200 + 15q$.

Find the inverse function and interpret its meaning.

- 5.14** Add, subtract and multiply two functions given in the form $y_1 = f_1(x)$ and $y_2 = f_2(x)$; write down, without simplification, the mathematical form for one function divided by another.

- 5.15** Understand the concept of a composite function and use the notation $y = f(g(x))$.

- 5.16** Deconstruct a composite function into its constituent functions, using inverse functions.

Find the inverse function of $f(x) = 5x - 8$.

Starting from the function $y = x$, describe how the function $y = (5x - 3)^2$ is constructed. Show how to deconstruct this function back to the original function.

Transformation of functions

- 5.17** Understand the transformations of the function $y = f(x)$ given by:

- $y = f(x) + a$, representing a translation by a in the positive y -direction;
- $y = f(x + a)$, representing a translation by $-a$ in the positive x -direction;
- $y = af(x)$, representing a stretch with scale factor a parallel to the y -axis;
- $y = f(ax)$, representing a stretch with scale factor $1/a$ parallel to the x -axis.

Use these and combinations of these transformations to sketch, stage by stage, the transformation of the graph of $y = f(x)$ into the graph of the transformed function.

The straight line $y = mx + c$ is the straight line $y = mx$ translated parallel to itself a distance c in the y -direction. When $y = mx$, the variable y is directly proportional to the variable x . By redefining the origin to the point $(0, c)$ the straight line $y = mx + c$ implies that the variable $(y - c)$ is directly proportional to the variable x , since $y - c = mx$ and this passes through the point $(0, c)$.

A function is defined by $f(x) = x^2$. Describe the functions a. $f(x - 2)$ and b. $f(x + 1)$, stating how the graphs of each function relate to the graph of $y = f(x)$, and give the defining equation for each function.

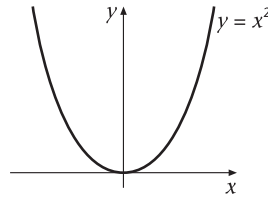
Modelling with circular functions

Examples could include oscillations on a spring, bungee jumping, pulse rate, blood pressure, alternating currents, daylight hours.

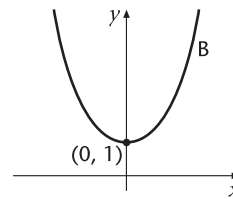
Composite functions

Use a 'function machine' to introduce a composite function and its inverse.

The diagram shows the graph with equation $y = x^2$.
On the same axes, sketch the graph with equation $y = 2x^2$.



Curve B is the translation, one unit up the y-axis, of $y = x^2$.
What is the equation of curve B?



Translate curve B two units to the left.
What is the equation of this new curve?

Transform the curve $y = x^3$ into the curve $y = 5x^3$. Describe the effect of the transformation. The curve is then translated one unit in the positive x-direction. What is the equation of this new curve?

Describe in words how the graph of $y = 1/x$ is transformed into the graph $y = 4 + 5/x$. Sketch each graph on the same set of axes.

Explain the difference between a. the functions $y = \cos x^\circ$ and $y = \cos(x + 45)^\circ$ and b. the functions $y = \cos x^\circ$ and $y = 2 \cos x^\circ$.

Modelling with exponential functions

5.18

Understand the ideas of exponential growth and decay and the forms of the associated graphs $y = a^x$, where $a > 0$; use a graphics calculator to plot the graphs of the exponential function, e^x , and the natural logarithm function, $\ln x$; know that one is the inverse function of the other and use this to find solutions to physical problems; solve for x the equation $y = a^x$ and use this in problems; use the log function (logarithm in base 10) on a calculator.

The growth of the Internet since 1990 has been modelled by the function $N = 0.2(1.8)^t$, where N is the number of users, counted in millions, t years from 1990. Plot the graph of this function. How many Internet users does the model predict for the year 2006?

When living organisms die the amount of carbon-14 present in the dead matter decays exponentially according to the formula $N = N_0 e^{-0.000121t}$, where N_0 is the initial quantity and t is the time in years. A bone uncovered at an archaeological site has 35% of its original carbon-14. Estimate the age of the bone. After how many more years will the bone have only 25% of its carbon-14?

The number of bacteria in a colony grows exponentially. At 1300 hours today the number of bacteria was 1000 and at 1500 hours it was 4000. How many bacteria were there at 1800 hours today? How many bacteria will there be at 1000 hours tomorrow?

The Global Report estimated the population of the world in 1975 as 4.1 billion people and that it was growing at the rate of 2% per year. Set up an equation to predict the world population t years from 1975. Use this model to predict the world's population in 2020. Discuss any assumptions you have made.

Earthquakes produce oscillations in the ground. The strength, S , of the quake is measured on the Richter scale and is given by $S = \log A$, where A is the measured amplitude of the oscillation as measured in millimetres on a calibrated seismograph. What amplitude of oscillation corresponds to a major earthquake with a Richter scale value of 7.8? What is the Richter scale value of an earthquake with an oscillation that has an amplitude of 2000 mm?

Modelling with exponential functions

Use examples of population growth and decay, cooling, radioactivity, spread of an epidemic, drug absorption, and compound interest.

Use a graphics calculator to support modelling.

6 Understand the concept of a limit

6.1 Understand the concept of a limiting value; for example, explain that:

- the sequence of terms $2, 4, 9, 25, \dots, n^2, \dots$ gets larger and larger as n gets larger and larger and that this sequence diverges as n tends to infinity;
- the sequence of terms $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \dots, \frac{n}{n+1}, \dots$ gets closer and closer to 1 as n gets larger and larger, and that this sequence converges to 1, the limit of the sequence, as n tends to infinity;
- the function $f(x) = \frac{1}{x}$ tends to minus infinity as x tends to zero from the negative side, but tends to positive infinity as x tends to zero from the positive side. At $x = 0$ the function is not defined. When x tends to infinity the function tends to zero from the positive side. When x tends to minus infinity the function tends to zero from the negative side. The lines $x = 0$ and $y = 0$ are asymptotes of the function;
- the function $f(x) = x$ tends to 2 when x tends to 2 from just below 2 and also tends to 2 when x tends to 2 from just above 2. For this function we can write $\lim_{x \rightarrow 2} f(x) = 2$.

The function $f(x) = \frac{x^2 - 1}{x + 1}$ is defined for all x except $x = -1$, and for these values is the same as the function $g(x) = x - 1$. Explain why this is so. For each function find the limit as x tends to -1 . What, if any, is the distinction between the functions f and g ?

The value of $\lim_{h \rightarrow 0} \frac{\sqrt{2+h} - \sqrt{2}}{h}$ is

- A. 0 B. $\frac{1}{2\sqrt{2}}$ C. $\frac{1}{2}$ D. $\frac{1}{\sqrt{2}}$ E. ∞

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6.2 Consider a chord across the graph of the function $y = f(x)$ between the points with coordinates $(x, f(x))$ and $(x + h, f(x + h))$, and show that if θ is the angle this chord makes with the positive x -axis then

$$\tan \theta = \frac{f(x+h) - f(x)}{(x+h) - x} = \frac{f(x+h) - f(x)}{h}$$

6.3 Evaluate $\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ for the following functions:

$$f(x) = x^2$$

$$f(x) = x^3$$

$$f(x) = x^{-1}$$

6.4 Understand that the limit $\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ for the function $y = f(x)$ is

represented geometrically as the slope of the tangent to the curve for this function at the point $(x, f(x))$.

Discuss how this limiting process corresponds to finding the slopes of chords drawn across a smaller and smaller interval of the curve from this point, and that in this process the chord itself tends to the tangent line at the point.

Limits

Include discussion of

$$\lim_{n \rightarrow 0} \left(1 + \frac{1}{n}\right)^n$$

It could be argued that this is equal to 1, since $\frac{1}{n}$ tends to zero as n gets larger and larger. On the other hand, it could be argued that $1 + \frac{1}{n}$ is always greater than 1 and so, taken to power n , gets larger and larger as n increases. Neither argument is correct. In fact, the expression tends to the irrational value $e = 2.7182818\dots$, the basis for the exponential function and the number base for natural logarithms.

7 Calculate the derivative of a function

7.1 Identify the tangent at the point $(x, f(x))$ on the function $y = f(x)$ as the derivative of the function at this point, and denote the derivative by either of the two common notations $\frac{dy}{dx}$ and $f'(x)$; interpret the derivative as a rate of change.

7.2 Calculate derivatives of simple powers of x from first principles, using the definition in NA 6.4 above.

Find $f'(x)$ for $f(x) = x^2$.

How does the mathematics change if $f(x) = 3x^2$?

Find $f'(x)$ for $f(x) = x^n$.

Use the definition of the derivative applied to this function and the binomial expansion for $(x + h)^n$, remembering that, since h is itself very small, h^2 and higher powers of h are negligibly small.

7.3 Know the general result that if $f(x) = ax^n$, where a is constant, then $f'(x) = anx^{n-1}$ for all real values of n .

The force of gravitational attraction between the Earth of mass M and a satellite of mass m is given by Newton's law of gravitation as $F = GMm/r^2$, where r is the distance between the centre of the Earth and the satellite and G is the universal constant of gravitation. Find $F'(r)$ and interpret its meaning.

7.4 Know that if $f(x) = ax^n \pm bx^m$, where a and b are constants, then $f'(x) = anx^{n-1} \pm bmx^{m-1}$.

Reading derivatives

Read the derivative dy/dx as 'dee-y by dee-x' or as 'the derivative of y with respect to x ' and the form $f'(x)$ as 'f-dash of x '.

Differentiation

The process of forming the derivative is called *differentiation*, so named because the method involves setting up a difference equation.

Geometry and measures

By the end of Grade 11, students use geometry and trigonometry to solve practical and theoretical problems. They know and use the sine and cosine rules, and calculate the area of a triangle using $\frac{1}{2} ab \sin C$. They use Pythagoras' theorem to show that $\sin^2 \theta + \cos^2 \theta = 1$ for any angle θ . They find the points of intersection of a straight line with a circle. They plot the graphs of circular functions and solve simple problems modelled by these functions. They prove standard circle theorems. They draw and use plans and elevations. They translate, reflect, rotate and enlarge two-dimensional geometric objects. They begin to use vectors to solve physical problems. They solve a range of problems involving compound measures, using appropriate units and dimensions. They explore geometry using ICT.

Geometry and measures

Students should develop an appreciation of the importance and range of geometrical applications in the real world, and the aesthetic qualities of geometric models. They should understand the nature and place of geometric reasoning and proof, and how geometry may be related to algebraic concepts, and vice versa.

Students should:

8 Develop geometrical reasoning and proof, and solve geometric problems

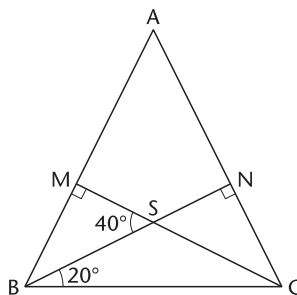
- 8.1 Use a dynamic geometry system to conjecture results and to explore geometric proof.

In triangle ABC, the altitudes BN and CM of the triangle ABC intersect at S.

$\angle MSB$ is 40° and $\angle SBC$ is 20° .

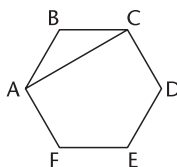
Prove that triangle ABC is an isosceles triangle.

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Each side of the regular hexagon ABCDEF is 10 cm long. Find the length of the diagonal AC.

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Use of ICT

Geometry is enhanced with use of a dynamic geometry system, or DGS, which provides an interactive means for investigating and hypothesising results that can then be proved as theorems.

Trigonometry, Pythagoras' theorem and the solution of triangles

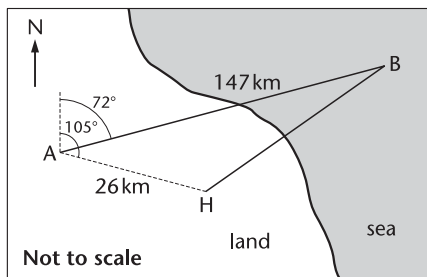
- 8.2 Know and use the sine rule and the cosine rule to solve triangles.

Show that Pythagoras' theorem is a special case of the cosine rule.

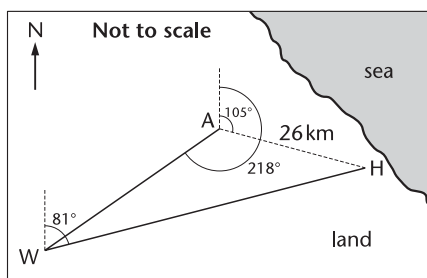
A triangle has its three angles in the ratio 2 : 3 : 4. Find to two significant figures the ratio of the lengths of its sides.

A ship sails 50 kilometres in a direction 032° and then 29 kilometres in a direction 315° . How far is the ship from its starting point? What is its bearing from its starting point?

A helicopter at airfield A received a distress call from a boat. The position of the boat, B, was given as 147 km from the airfield, on a bearing of 072°. A man on the boat is flown to hospital. Calculate the distance the helicopter travelled from the boat to the hospital at H.

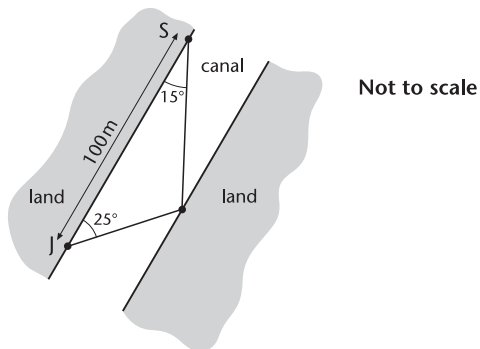


On another occasion the helicopter travelled from the airfield on a bearing of 218° to fly a pregnant woman at W to the hospital. The helicopter then flew on a bearing of 081° to the hospital, H. Calculate the distance the helicopter travelled from where it picked up the woman to the hospital.



8.3 Solve triangle problems in two and three dimensions.

The two sides of a canal are straight, parallel and the same height above the water level. Jana and Sharifa want to find the width of the canal. They measure 100 m on the canal bank and stand facing each other at the points J and S. Jana measures the angle she turns through to look at the post, P, as 25°. Sharifa measures the angle she turns through to look at the post as 15°. Calculate the width of the canal.



The Great Pyramid of Cheops in Egypt is built on a square base with side 230 metres. Each face of the pyramid is at 52° to the horizontal. Calculate the height of the pyramid. Calculate the inclination of an edge of the pyramid to the horizontal.



The Great Pyramid of Cheops at Giza. Source: www.kingtutshop.com

Triangle problems

Include the terminology *angle of inclination* and *angle of declination*.

- 8.4** Calculate the area of a triangle using $\frac{1}{2}ab \sin C$.
- 8.5** Find the points of intersection of a straight line with a circle by using algebraic substitution from the equation of the straight line into the equation of the circle.

Find the points where the line $4x - 3y = 0$ cuts the circle $x^2 + y^2 = 100$.

Circular functions

- 8.6** Use the unit circle $x^2 + y^2 = 1$ to plot and describe the features of the graphs of the circular functions

$$f(\theta) = \sin \theta$$

$$f(\theta) = \cos \theta$$

$$f(\theta) = \tan \theta$$

where θ is measured in radians and $0 \leq \theta \leq 2\pi$; know the symmetries and periodicities of these functions; know that any point on the unit circle has coordinates $(\cos \theta, \sin \theta)$, where θ is the angle the radius to the point makes with the positive x -axis.

Explain why $\sin(2\pi - \theta) = \sin \theta$.

Give the exact value of $\cos \frac{5\pi}{4}$. What other angle has the same cosine value?

Draw the tangent line at the origin on the curve $y = \sin x$. Use this to estimate the value of $\sin x$ for small values of x . Explain why it is important to use radian measure in this context.

- 8.7** Use Pythagoras' theorem to show that $\sin^2 \theta^\circ + \cos^2 \theta^\circ = 1$ for any angle θ° .

Verify this result for the angles 30° , 45° and 60° . What happens when $\theta^\circ = 90^\circ$?

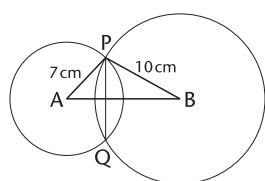
- 8.8** Solve simple problems modelled by circular functions.

Circle geometry

- 8.9** Prove the circle theorems:

- The angle subtended by an arc at the centre of the circle is twice the angle subtended by the arc at a point on the circle, including, as a special case, the angle in semicircle is a right angle.
- Angles in the same segment subtended by a chord are equal.
- The angle subtended by a chord at the centre of a circle is twice the angle between the chord and the tangent to the circle at an end point of the chord.
- When two chords BC and DE in a circle intersect at A then $AB \times AC = AD \times DE$.
- Opposite angles of a cyclic quadrilateral are supplementary.

Two circles with centres at A and B have radii of 7 cm and 10 cm as shown in the diagram. The length of the common chord PQ is 8 cm.



Calculate the length of AB.

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Trigonometric functions

The full names of these functions are sine, cosine and tangent. For the calculus of these functions it will be necessary to work with the reciprocal functions (sec), cosecant (cosec) and cotangent (cot).

Include use of the term *amplitude*.

Trigonometric identities

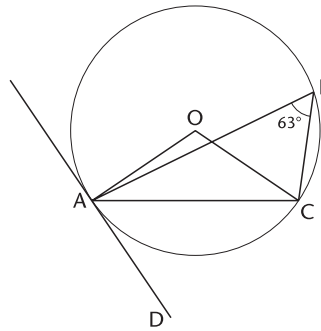
Include deriving simple related identities.

Circle theorems

Include terms associated with a circle: *inscribed circle, circumcircle, cyclic quadrilateral*.

Include the use of a dynamic geometry system (DGS).

In the diagram on the right, AD is a tangent to the circle with centre O .
 $\angle ABC$ is 63° and AC is a chord of the circle.
 AB is 18 cm and BC is 3 cm.



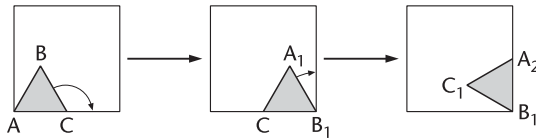
Calculate the values of $\angle AOC$, $\angle OCA$ and $\angle CAD$.
 Calculate the area of triangle ABC .
 Calculate the length of AC .

9 Work with transformations and projections

9.1

Transform rectilinear figures using combinations of translations, rotations about a centre of rotation, enlargements about a centre of enlargement, and reflections about a line; understand the meanings of positive, negative and fractional scale factors in enlargements.

An equilateral triangle ABC has side length 10 cm. It rotates around the inside of a square of side length 20 cm.

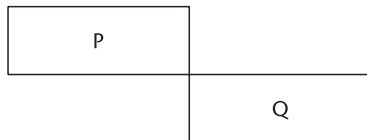


- Triangle ABC rotates about C to the position shown as CA_1B_1 . What is the angle of rotation?
- Calculate the distance along the path travelled by point A in turning from A to A_1 .
- Calculate the distance along the path travelled by point A in turning from A_1 to A_2 .
- The triangle continues rotating around the inside of the square in the same way until it is back at the original position. Which of the original points A , B or C will point A land on when it has completed its rotations around the inside of the square?

The rectangle Q in the diagram below CANNOT be obtained from the rectangle P by means of a

- reflection about an axis in the plane of the page
- rotation in the plane of the page
- translation
- translation followed by a reflection

Circle the correct answer.



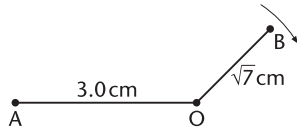
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A triangle has vertices at the points $(4, 5)$, $(6, 1)$ and $(8, 11)$. The triangle is enlarged by a factor of 2 about a centre of enlargement at the point $(3, -3)$. Draw the enlarged triangle in its correct position on a coordinate grid.

Transformations

Transformations are best developed through use of a dynamic geometry system (DGS).

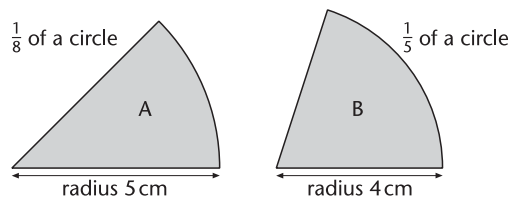
The line segment OA is 3.0 cm long. The line segment OB is $\sqrt{7}$ cm long.
 OB can rotate in a horizontal plane about the point O .



- Find the maximum possible distance B can be from A . Explain whether your answer is a rational number or an irrational number.
- Find the minimum possible distance B can be from A . Explain whether your answer is a rational number or an irrational number.
- Sketch a different position for the line segment OB so that the distance from A to B , AB , is a rational number. Confirm by calculation that your answer is a rational number.
- OB is reduced in length to 2.6 cm. OA is still 3.0 cm long. Calculate the distance AB when angle AOB is 120° .
- The lengths of 2.6 cm and 3.0 cm are accurate to one decimal place. The 120° angle is accurate to the nearest degree. Calculate the greatest and least possible values of AB .

9.2

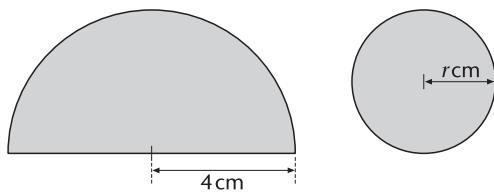
Visualise the effect of transformations on a plane figure; know that the image of a planar figure under rotation or reflection is congruent to the original figure before rotation or reflection and that every circle is similar to any other circle.



The diagram shows parts of two circles, sector A and sector B .

- Which sector has the bigger area?
- The perimeter of a sector is made from two straight lines and an arc. Which sector has the bigger perimeter?

A semicircle, of radius 4 cm, has the same area as a complete circle of radius r cm.
 What is the radius of the complete circle?



Not drawn to scale

9.3

Draw the plan and elevation of three-dimensional rectilinear objects.

10 Use vectors

- 10.1** Consider coordinate systems as grids for moving around space in two or three dimensions; understand *position vector*, *unit vector* and *components* of a vector.

A particle is at the point (6, 2). What is its position vector in terms of the unit vectors \mathbf{i} and \mathbf{j} in the x - and y -directions respectively? Calculate the length (magnitude) of this vector.

- 10.2** Interpret a translation as a vector displacement; know that a vector displacement from A to B depends only on the starting point A and the finish point B and not on intermediate steps from A to C to D to ... to B, and that the vector sum of all these separate displacements from A to B is equivalent to the resultant displacement from A to B directly.

- 10.3** Add and subtract two vectors in up to three dimensions and draw corresponding vector diagrams.

Four vectors \mathbf{a} , \mathbf{b} , \mathbf{c} and \mathbf{d} are given by $\mathbf{a} = 2\mathbf{i} - 3\mathbf{j}$, $\mathbf{b} = 5\mathbf{j} + \mathbf{k}$, $\mathbf{c} = 4\mathbf{i} - 7\mathbf{k}$ and $\mathbf{d} = 3\mathbf{i} + \mathbf{j}$. Find $\mathbf{a} + \mathbf{b}$, $\mathbf{b} - \mathbf{c}$, $\mathbf{a} - \mathbf{b} - \mathbf{c}$. Draw vector diagrams to represent $\mathbf{a} + \mathbf{d}$ and $\mathbf{a} - \mathbf{d}$. What are the components of these two vectors in the \mathbf{i} and \mathbf{j} directions?

- 10.4** Multiply a vector by a scalar and know that this amounts to stretching the vector; calculate the magnitude and direction of a vector; use vectors to calculate displacement and velocity in a range of contexts.

A particle moves with constant velocity from A to B. Its position vector at A is $\mathbf{a} = \mathbf{i} + \mathbf{j}$ and its position vector at B is $\mathbf{b} = 5\mathbf{i} - 7\mathbf{j}$. Calculate the vector displacement from A to B. If distance is measured in metres, show that the distance from A to B is $4\sqrt{5}$ metres. The particle takes 2 seconds to move from A to B. What is its velocity?

- 10.5** Use the scalar product of two vectors to calculate the angle between the vectors and the scalar product of a vector with itself to find the magnitude of the vector.

Find the magnitude of each of the vectors \mathbf{a} and \mathbf{d} in the example in GM 10.3 above. Calculate the angles between these vectors.

- 10.6** Solve physical problems using vectors.

\mathbf{i} and \mathbf{j} are unit vectors in the east and north direction respectively. A ship has position $\mathbf{r} = 3\mathbf{i} + 4\mathbf{j}$ at 1200 hours. It then moves with constant velocity $\mathbf{v} = 4\mathbf{i} - 5\mathbf{j}$. The velocity is measured in kilometres per hour. What is the speed of the ship (the magnitude of its velocity)? What is the position of the ship at 1500 hours?

A particle of mass m kilograms is moving with constant acceleration \mathbf{a} , measured in metres per second per second. The total external force \mathbf{F} acting on the particle is measured in newtons, and is the vector sum of the individual forces acting on the particle. The relationship between \mathbf{F} and \mathbf{a} is given by Newton's second law of motion and is $\mathbf{F} = m\mathbf{a}$.

A particle of mass 2 kg is acted upon by two forces $\mathbf{F}_1 = \mathbf{i} - \mathbf{j}$ and $\mathbf{F}_2 = 3\mathbf{j}$. Find the acceleration of the particle and give its magnitude.

11 Use a range of measures and compound measures to solve problems

- 11.1** Calculate lengths, areas and volumes of geometrical shapes.

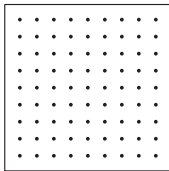
Vectors

In three dimensions, vectors provide a natural language to place and displace figures in space. They also link with Cartesian coordinate systems.

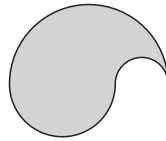
Unit vectors

Unit vectors in three mutually perpendicular directions are usually written as \mathbf{i} , \mathbf{j} and \mathbf{k} .

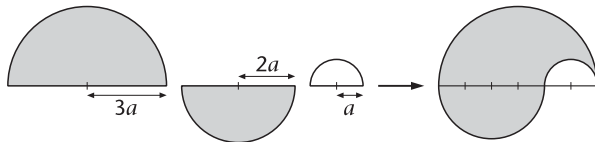
On the pinboard, draw a trapezium that has a perimeter of $6 + 4\sqrt{2}$.



This shape is designed using three semicircles.

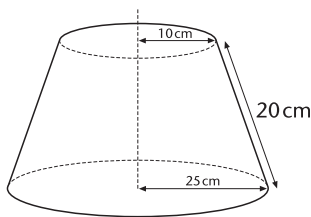


The radii of the semicircles are $3a$, $2a$ and a .



- Find the area of each semicircle, in terms of a and π , and show that the total area of the shape is $6\pi a^2$.
- Find a when the area is 12 cm^2 , leaving your answer in terms of π .

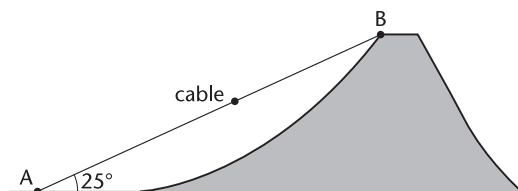
A light shade is in the form of a frustum of a right cone. The radius at the top of the shade is 10 cm and the radius at the bottom is 25 cm . Find the surface area of the material used for the light shade.



- 11.2** Work with compound measures including density, average speed and acceleration, measures of rate (such as rate of growth of income), and population density (number of people per unit area), using appropriate units and dimensions.

Jabor sees a flash of lightning. 25 seconds later he hears the sound of thunder.
 Speed of light is about $1.1 \times 10^9 \text{ km per hour}$.
 Speed of sound is about $1.2 \times 10^3 \text{ km per hour}$.
 Calculate how far away Jabor is from the lightning.

A cable car takes passengers to the top of a volcano. It starts from station A and takes 16 minutes to reach station B at the top of the volcano. The average speed of the cable car is 2 metres per second. The cable car is at an angle 25° to the horizontal. Find, to the nearest metre, the height of the volcano as measured from A.



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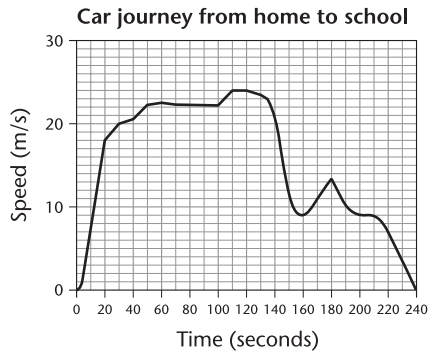
Compound measures

Reinforce links with science and technology, using power or rate of doing work, momentum, average speed and acceleration.

Rates

Students who will study quantitative methods in Grade 12 should work with a range of rates to support later work in probability and statistics.

Wafa recorded the speed of a car every 10 seconds throughout a short journey from her home to school. She used the data to draw a speed–time graph.



- Find a point during the journey when the car's speed was increasing most quickly. Mark this point as *P*. By drawing appropriate lines on the graph, find the acceleration of the car at point *P*.
- Find a point during the journey when the car's speed was decreasing most quickly. Mark this point as *Q*. By drawing appropriate lines on the graph, find the acceleration of the car at point *Q*.
- The car uses least fuel when it travels at speeds between 20 m/s and 25 m/s. Find an approximate value for the area under the graph for the period when the car was travelling at between 20 m/s and 25 m/s. What does this area represent? Give the correct units.

Probability and statistics

By the end of Grade 11, students use a simple mathematical model to calculate, for a particular set of events, the theoretical probability of obtaining a particular outcome for a random variable associated with those events. They calculate probabilities of single and combined events, and understand risk as the probability of the occurrence of an adverse event. They use tree diagrams to represent and calculate the probabilities of compound events when the events are independent and when one event is conditional on another. They use simple simulations and consider trends over time using a moving average. They analyse results to draw conclusions and use a range of graphs, charts and tables to present their findings. They use relevant statistical functions on a calculator and ICT applications to present statistical tables and graphs.

Probability and statistics

Students should know that statistics is the branch of mathematics used to predict the outcomes of large numbers of events when these outcomes are uncertain, and that probability lies at the heart of statistics. They should be aware of the uses of statistics in society and recognise when statistics are used sensibly and when they are misused or likely to be misunderstood.

Students should:

12 Understand random variables and calculate probability

12.1 Know that all probability values lie between 0 and 1, and that the extreme values correspond respectively to impossibility and certainty of occurrence.

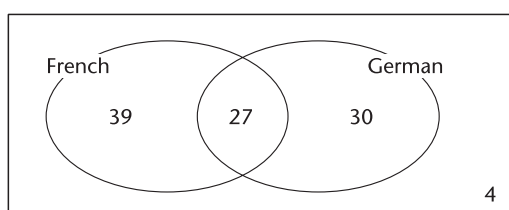
12.2 Understand that a random variable has a range of values that cannot be predicted with certainty, and investigate common examples of random variables; measure the empirical probability (relative frequency) of obtaining a particular value of a random variable.

- 12.3** Use a simple mathematical model to calculate, for a particular set of events, the theoretical probability of obtaining a particular outcome for a random variable associated with those events.

A company makes computer disks. It tested a random sample of disks from a large batch. The company calculated the probability of any disk being defective as 0.025. Naima buys two disks.

- Calculate the probability that both disks are defective.*
- Calculate the probability that only one of the disks is defective.*
- The company found three defective disks in the sample they tested. How many disks were likely to have been tested?*

100 students were asked whether they studied French or German. 27 students studied both French and German.



- What is the probability that a student chosen at random will study only one of the languages?*
- What is the probability that a student who is studying German is also studying French?*

Assume that it is equally likely for a woman to give birth to a girl as it is to give birth to a boy. What is the probability that a woman with six children has four girls and two boys? What is the probability that if another woman has four children they are all boys? A woman with three daughters is going to have a fourth child. What is the probability that the fourth child will be boy?

A piece of string is 12 centimetres long. It is randomly cut into two pieces. What is the probability that one piece has length greater than 9 cm?

- 12.4** Understand *risk* as the probability of occurrence of an adverse event; investigate some instances of risk in everyday situations, including in insurance and in medical and genetic matters.

The probability of dying of cancer is $\frac{1}{3}$. What is the probability that if three people are chosen at random two of them will die of cancer? What is the probability that none of them will die of cancer?

Probability of combined events

- 12.5** Understand when two events are *mutually exclusive*, and when a set of events is *exhaustive*; know that the sum of probabilities for all outcomes of a set of mutually exclusive and exhaustive events is 1, and use this in probability calculations.

- 12.6** Know that when two events A and B are mutually exclusive the probability of A or B, denoted by $P(A \cup B)$, is $P(A) + P(B)$, where $P(A)$ is the probability of event A alone and $P(B)$ is the probability of event B alone.

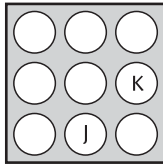
Mona has a chance of 1 in 4 of passing on a particular genetic condition to one of her children. Mona has three children. Calculate the probability that two of the children will inherit the condition. What is the probability that none of her children will inherit the condition? What is the probability that at least one of her children will inherit the condition?

Distributions

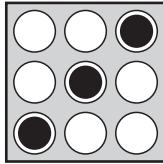
Include as mathematical models the rectangular distribution and the binomial distribution, and simple applications of these.

A computer game has nine circles arranged in a square. The computer chooses circles at random and shades them black.

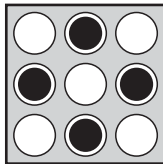
- a. At the start of the game, two circles are to be shaded black. Show that the probability that both circles J and K will be shaded black is $\frac{1}{36}$.



- b. Halfway through the game, three circles are to be shaded black. Here is one example of the three circles shaded black in a straight line. Show that the probability that the three circles shaded black will be in a straight line is $\frac{8}{84}$.



- c. At the end of the game, four circles are to be shaded black. Here is one example of the four circles shaded black forming a square. What is the probability that the four circles shaded black form a square?



12.7

Know that two events A and B are *independent* if the probability of A and B occurring together, denoted by $P(A \cap B)$, is the product $P(A) \times P(B)$.

In a class of 35 students, the probability that a student picked at random is taller than 1.8 metres is 0.2 and the probability that the student wears spectacles is 0.3.

What is the probability when three students are chosen at random that two are over 1.8 metres in height and that one of them wears spectacles?

Show that two events A and B are mutually exclusive when $P(A \cap B) = 0$.

On a road there are two sets of traffic lights. The traffic lights work independently.

For each set of traffic lights, the probability that a driver will have to stop is 0.7.

- a. A woman is going to drive along the road. What is the probability that she will have to stop at both sets of traffic lights? What is the probability that she will have to stop at only one of the two sets of traffic lights?
- b. In one year, a man drives 200 times along the road. Calculate an estimate of the number of times he drives through both sets of traffic lights without stopping.

Adel makes two clay pots. Each pot is fired independently. The probability that a pot cracks while being fired is 0.03.

- a. Calculate the probability that **both** of Adel's pots crack while being fired.
- b. Calculate the probability that **only one** of Adel's pots cracks while being fired.
- c. Adel has enough clay for 80 pots. He receives an order for 75 pots. Does he have enough clay to make 75 pots without cracks? Explain your answer.

A warning system installation consists of two independent alarms having probabilities of operating in an emergency of 0.95 and 0.90 respectively. Find the probability that at least one alarm operates in an emergency.

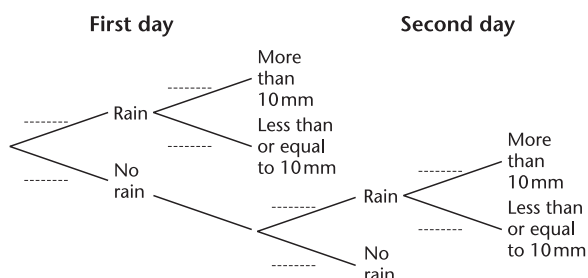
- A. 0.995 B. 0.975 C. 0.95 D. 0.90 E. 0.885

TIMSS Grade 12

- 12.8** Use tree diagrams to represent and calculate the probabilities of compound events when the events are independent and when one event is conditional on another.

On a tropical island the probability of it raining on the first day of the rainy season is $\frac{2}{5}$. If it does not rain on the first day, the probability of it raining on the second day is $\frac{7}{10}$. If it rains on the first day, the probability of it raining more than 10 mm on the first day is $\frac{1}{5}$. If it rains on the second day but not on the first day, the probability of it raining more than 10 mm is $\frac{1}{4}$.

You may find it helpful to fill in this tree diagram before answering the questions below.



- What is the probability that it rains more than 10 mm on the second day, and does not rain on the first?
- What is the probability that it has rained by the end of the second day of the rainy season?
- Why is it not possible to work out the probability of rain on both days from the information given?

20 per cent of the population of a country has a particular disease. A test can be given to help determine whether or not people have the disease. The probability that the test is positive for those that have the disease is 0.7. But there is a 0.1 chance that a patient who does not have the disease registers positive on the test.

- Find the probability that an individual selected at random tests positive, but does not have the disease.
- Another person is chosen at random. Calculate the probability that the test result for this person is positive.

- 12.9** Know that in general if event B is dependent on event A, then the probability of A and B both occurring is $P(A \cap B) = P(A) \times P(B|A)$, where $P(B|A)$ is the conditional probability of B given that A has occurred.

13 Calculate moving averages

- 13.1** Consider trends over time and calculate a moving average.

Find out about the cost, in Qatar, of a barrel of crude oil over the period January 2000 to March 2004. Analyse the data over periods of three months and compare the moving average price per barrel. Discuss your findings.

14 Simulation

- 14.1** Use coins, dice or random numbers to generate models of random data.

Reema passes through two sets of traffic lights each morning on her way to work. She has timed both of them.

- Reema has found that the first set shows green for 30 out of 50 seconds. What is the probability that Reema does not stop at the first set of traffic lights? Describe a method for simulating this probability using random numbers.

Random numbers

These can be generated on a scientific or graphics calculator using the RND and RAN function keys.

- b. *The second set of traffic lights shows green for 30 out of 90 seconds. What is the probability that Reema does not stop at the second set of traffic lights. Describe a method for simulating this probability using random numbers.*
- c. *To simulate Reema's journey through both sets of traffic lights you must choose two random numbers. Repeat this simulation 20 times. Use the simulation to estimate the probability that Reema will have to stop:*
- *at both sets of traffic lights;*
 - *at only one set of traffic lights;*
 - *at neither set of traffic lights.*

Do an investigation using random numbers to simulate waiting times in a doctor's surgery.

15 Use of ICT

- 15.1** Use a calculator with statistical functions to aid the analysis of large data sets, and ICT packages to present statistical tables and graphs.
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ICT opportunity

A range of applications can support data handling.