Mathematics standards

Grade 11
Foundation

Summary of students' performance by the end of Grade 11

Reasoning and problem solving

Students solve routine and non-routine problems in a range of mathematical and other contexts, and use mathematics to model and predict the outcomes of real-world applications. They identify and use connections between mathematical topics. They break down complex problems into smaller tasks, and set up and perform appropriate manipulations and calculations. They develop and explain chains of logical reasoning, using correct mathematical notation and terms. They generate mathematical proofs and identify exceptional cases. They aim to generalise. They approach problems systematically, knowing when it is important to enumerate all outcomes. They conjecture alternative possibilities with 'What if …?' and 'What if not …?' questions. They synthesise, present, interpret and criticise mathematical information, working to expected degrees of accuracy. They recognise when to use ICT and do so efficiently.

Number and algebra

Students use the laws of exponents, proportional reasoning and harder percentage calculations to solve problems, including compound interest problems. They find the sums of geometric sequences and convert any recurring decimal to a fraction. They simplify and combine numeric and algebraic fractions and multiply any two monomial, binomial or trinomial expressions, collecting and simplifying similar terms. They factorise quadratic expressions, relating the factorisations to geometric representations. They generate formulae from physical contexts and rearrange formulae connecting two or more variables. Through their continued study of linear, quadratic, reciprocal and other functions and their graphs, and the solution of associated equations, students appreciate a range of numerical and algebraic applications in the real world. They solve simple problems represented by regions of linear inequality. They recognise when quadratic functions are increasing, decreasing or stationary. They model situations with quadratic functions and find exact and approximate solutions of quadratic equations, and a pair of simultaneous equations, one linear and one quadratic. They solve problems involving inverse proportion. They find the tangent at a point on the graph of a function. They continue to use realistic data and ICT to analyse problems.

Geometry and measures

Students continue to use their knowledge of geometry and trigonometry to solve practical and theoretical problems relating to shape and space. They solve right-angled triangles in two and three dimensions using the standard trigonometric ratios. They know and use the sine rule and cosine rule, and calculate the area of a triangle using $\frac{1}{2}ab\sin C$. They use Pythagoras' theorem to show that $\sin^2\theta + \cos^2\theta = 1$ for any angle θ, to find the distance between two points and to set up the Cartesian equation of a circle. They find the points of intersection of a straight line with a circle. They plot the graphs of circular functions and solve simple problems modelled by these functions. They prove standard circle theorems. They continue to use SI units and a range of measures to solve problems, including radian measure

to calculate sector areas and arc lengths and compound measures. They use bearings, latitude, longitude and great circles to solve problems relating to position, distance and displacement on the Earth's surface. They use ICT to explore geometry.

Probability and statistics

Students plan questionnaires and surveys to collect meaningful primary data from samples. They know the importance of representative samples, and can locate sources of bias. They collect data from secondary sources, including the Internet, and ask and answer questions related to the data. They group data and plot histograms and other frequency and relative frequency distributions. They draw stem-and-leaf diagrams and box-and-whisker plots. They continue to calculate and use measures of central tendency. They analyse results to draw conclusions and use a range of graphs, charts and tables to present their findings.

Content and assessment weightings for Grade 11

The foundation mathematics standards are grouped into four strands: reasoning and problem solving; number and algebra; geometry and measures; and probability and statistics.

The reasoning and problem solving strand cuts across the other three strands. Reasoning, generalisation and problem solving should be an integral part of the teaching and learning of mathematics in all lessons.

The weightings of the content strands relative to each other are as follows:

Foundation	Number and algebra	Geometry and measures*	Probability and statistics
Grade 10	55%	30%	15%
Grade 11	55%	30%	15%
Grade 12	50%	25%	25%

* including trigonometry

The standards are numbered for easy reference. Those in shaded rectangles, e.g. 1.2, are the performance standards for all foundation students. The national tests for foundation mathematics will be based on these standards.

Grade 11 teachers should review and consolidate Grade 10 standards where necessary.

Mathematics standards

Grade 11
Foundation

Reasoning and problem solving

By the end of Grade 11, students solve routine and non-routine problems in a range of mathematical and other contexts, and use mathematics to model and predict the outcomes of real-world applications. They identify and use connections between mathematical topics. They break down complex problems into smaller tasks, and set up and perform appropriate manipulations and calculations. They develop and explain chains of logical reasoning, using correct mathematical notation and terms. They generate mathematical proofs and identify exceptional cases. They aim to generalise. They approach problems systematically, knowing when it is important to enumerate all outcomes. They conjecture alternative possibilities with 'What if …?' and 'What if not …?' questions. They synthesise, present, interpret and criticise mathematical information, working to expected degrees of accuracy. They recognise when to use ICT and do so efficiently.

Students should:

1 Use mathematical reasoning to solve problems

1.1 Solve routine and non-routine problems in a range of mathematical and other contexts, including open-ended and closed problems.

1.2 Use mathematics to model and predict the outcomes of real-world applications; compare and contrast two or more given models of a particular situation.

1.3 Identify and use interconnections between mathematical topics.

1.4 Break down complex problems into smaller tasks.

1.5 Use a range of strategies to solve problems, including working the problem backwards and then redirecting the logic forwards; set up and solve relevant equations and perform appropriate calculations and manipulations; change the viewpoint or mathematical representation, and introduce numerical, algebraic, graphical, geometrical or statistical reasoning as necessary.

1.6 Develop chains of logical reasoning, using correct mathematical notation and terms.

1.7 Explain their reasoning, both orally and in writing.

1.8 Generate mathematical proofs, and identify exceptional case.

1.9 Aim to generalise.

1.10 Approach a problem systematically, recognising when it is important to enumerate all outcomes.

1.11 Conjecture alternative possibilities with 'What if …?' and 'What if not …?' questions.

1.12 Synthesise, present, discuss, interpret and criticise mathematical information presented in various mathematical forms.

Key standards

Key performance standards are shown in shaded rectangles, e.g. 1.2.

Cross-references

Standards are referred to using the notation RP for reasoning and problem solving, NA for number and algebra, GM for geometry and measures, and PS for probability and statistics, e.g. standard NA 2.3.

Examples of problems

The examples of problems in italics are intended to clarify the standards, not to represent the full range of possible problems.

Reasoning and problem solving

Reasoning, generalisation and problem solving should be an integral part of the teaching and learning of mathematics in all lessons.

Proofs

Relate to the mathematics in the other strands.

1.13 Work to expected degrees of accuracy, and know when an exact solution is appropriate.

1.14 Recognise when to use ICT and when not to, and use it efficiently.

Number and algebra

By the end of Grade 11, students use the laws of exponents, proportional reasoning and harder percentage calculations to solve problems, including compound interest problems. They find the sums of geometric sequences and convert any recurring decimal to a fraction. They simplify and combine numeric and algebraic fractions and multiply any two monomial, binomial or trinomial expressions, collecting and simplifying similar terms. They factorise quadratic expressions, relating the factorisations to geometric representations. They generate formulae from physical contexts and rearrange formulae connecting two or more variables. Through their continued study of linear, quadratic, reciprocal and other functions and their graphs, and the solution of associated equations, students appreciate a range of numerical and algebraic applications in the real world. They solve simple problems represented by regions of linear inequality. They recognise when quadratic functions are increasing, decreasing or stationary. They model situations with quadratic functions and find exact and approximate solutions of quadratic equations, and a pair of simultaneous equations, one linear and one quadratic. They solve problems involving inverse proportion. They find the tangent at a point on the graph of a function. They continue to use realistic data and ICT to analyse problems.

Algebra

Students should learn that algebra enables generalisation and the establishment of relationships between quantities and/or concepts. They should understand the nature and place of algebraic reasoning and proof, and how algebra may be related to geometric concepts, and vice versa.

Students should:

2 Identify and use number sets

2.1 Use as appropriate the language of number sets from Grade 10.

What is the solution set in $\mathbb{R}$ of the quadratic equation $4x^2 + 3x - 1 = 0$?

What is the solution set in $\mathbb{N}$ of the quadratic equation $(x - 2)(3x + 1) = 0$?

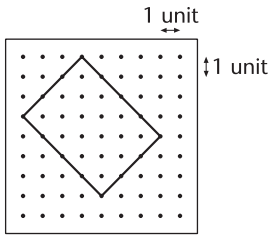
What is the complement set of the set $A \cup (B \cup C)$? Show this on a Venn diagram.

3 Use index notation and solve numerical problems

3.1 Understand and use the laws of exponents to calculate and simplify problems, including mental calculations in appropriate cases.

Calculate mentally the value of 9999×0.033.

In this question, you should not use a calculator.
An elastic band is fixed on four pins on a pinboard, as shown in the diagram below.
Show that the total length of the band in this position is $14\sqrt{2}$ units.



Laws of exponents

For $a > 0$:
$a^x \times a^y = a^{x+y}$
$a^x \div a^y = a^{x-y}$
$(a^x)^y = a^{xy}$
$(a^{1/n})^n = a$
$a^0 = 1$

Use standard form to estimate the value of $4350 \times 237.8 \times \pi^2$.

The Earth is approximately a sphere of radius 6378 kilometres.
Without using a calculator estimate the circumference at the equator.

The mass of the Earth is 5.98×10^{24} *kg. A typical man has a mass of about 70 kg.*
Approximately how many men would have a total mass equal to that of the Earth?

Light travels at about 300 000 kilometres per second.
Use standard form to find the distance away from the Earth of a light-emitting body whose light signal is received at Earth one year after it is emitted.

The Earth completes its orbit around the Sun in 365 days. The Earth is 148.8 million kilometres from the Sun. Assume that the Earth's orbit is circular and that it travels around the Sun with constant speed. Calculate the Earth's speed in kilometres per hour.

3.2 Solve a range of problems using the multiplicative nature of proportional reasoning.

3.3 Perform harder percentage calculations, including taking a percentage of a percentage, inverse percentage and compound interest problems.

Percentages

Include problems involving the annual equivalent rate (AER).

QR 10 000 has to be invested in deposit accounts. There is a choice of two accounts. One account pays an annual interest of 4.6%. The other account pays interest of 1.5% three times a year. What is the AER of the second account?
Which is the better account to invest in and how much more interest will there be after one year in this account than in the other account?

3.4 Investigate the problem of compounding interest more and more frequently and note that this tends to a limiting value; use this context to learn about the number e.

4 Generate and manipulate algebraic expressions and formulae, and solve algebraic equations

4.1 Know the properties of geometric sequences and the conditions under which an infinite geometric series can be summed.

Grains of rice are placed on each square of a chessboard. The board has 64 squares. One grain is placed on the first square, two on the second, four on the third, eight on the fourth, and so on. Calculate the total number of grains of rice on the chessboard.

1 kilogram of rice contains approximately 16 000 grains of rice.
Estimate the weight of all the rice on the chess board.

The sum of the infinite geometric series $1 - {}^1/_2 + {}^1/_4 - {}^1/_8 + \ldots$ *is*
A. ${}^5/_8$ B. ${}^2/_3$ C. ${}^3/_5$ D. ${}^3/_2$

Investigate compound interest problems as examples of geometric series.

4.2 Convert any recurring decimal to an exact fraction.

Explain why $0.\dot{1}\dot{2} = \frac{4}{33}$.

4.3 Develop further a sense of working with symbols, understanding that the transformation of all such algebraic objects generalises the well-defined rules of arithmetic, and knowing that letters are used to represent:

- the solution set of initially unknown numbers in *equations*;
- defined variables in *formulae*;
- generalised independent numbers in *identities*;
- new equations, expressions or functions defined in terms of known, or given, expressions or functions.

Is $(x - a)(x^2 + ax + a^2) = x^3 - a^3$ an equation or an identity? Discuss what happens to this mathematical statement when a is replaced throughout by –a.

Give examples of what is meant by an associative law and a distributive law.

Calculate $(\sqrt{5} + \sqrt{3})(\sqrt{5} - \sqrt{3})$.

4.4 Combine numeric or algebraic fractions, and multiply combinations of monomial, binomial and trinomial expressions, collecting and simplifying similar terms.

Use Pascal's triangle to read off the coefficients of the powers of x in the expansion of $(1 + x)^n$ for different values of the positive integer n.
Check the results for $n = 3$ by expanding $(1 + x)^3$.

Simplify $(2x - 3)(x^2 + x - 10)$.

4.5 Factorise expressions of the form $a^2x^2 - b^2y^2$, and quadratic expressions; conceptualise geometric representations for these factorisations and other similar quadratic expressions.

Quadratic expressions

Include the forms:

$x^2 + (\alpha + \beta)x + \alpha\beta$

$x^2 - (\alpha + \beta)x + \alpha\beta$

$x^2 \pm (\alpha - \beta)x - \alpha\beta$

Without using a calculator, find the exact value of $7.92^2 - 2.08^2$.

Explain why $(a + b)^2 \neq a^2 + b^2$.

Draw a diagram to represent the identity $(a + b)^2 = a^2 + 2ab + b^2$.

Draw a diagram to represent the identity $(a - b)^2 = a^2 - 2ab + b^2$.

Construct some quadratic expressions from two linear factors in a and b and draw geometric representations for them.

4.6 Simplify numeric and algebraic fraction expressions by cancelling common factors; rationalise a denominator of a fraction when the denominator contains simple combinations of surds.

Rationalise the expression $1/(\sqrt{2} + \sqrt{3})$.

4.7 Generate further formulae from a physical context, and rearrange formulae connecting two or more variables; substitute an expression for a given variable into a different formula containing this variable.

Melons cost QR 1.5 each and apples cost QR 3.75 per kilogram. A man buys apples and melons at the supermarket. Write a formula to describe the total cost of his purchase. Investigate how many melons and how many kilograms of apples he could buy for QR 30.

Find R in terms of R_1 and R_2 when $1/R = 1/R_1 + 1/R_2$.

The volume of a solid cylinder of length h and radius r is V.
Find a formula for the curved surface area, A, of the cylinder in terms of r and h.
Use this formula to find a formula expressing V in terms of A and r.

5 Generate and solve problems with functions and graphs

5.1 Use a graphics calculator to plot and interpret a range of functional relationships, some continuous and others discontinuous, arising in familiar contexts.

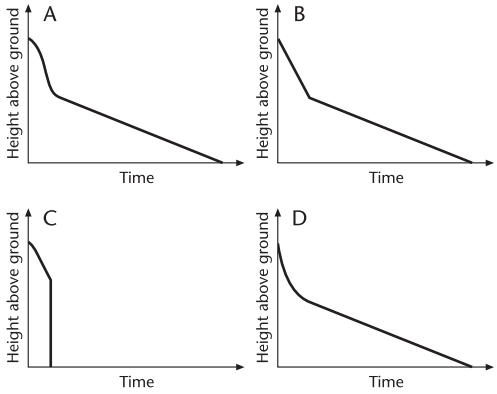
ICT opportunity

Graphics calculators or graph plotters can be used to explore a range of functional relationships.

Plot the graph of $y = 1/x^2$ for the domain set $\{x: x \in \mathbb{R} \text{ and } 1 \leq x \leq 4\}$.
Discuss whether the domain could be extended.

Plot the curve $y = \sqrt{x}$ on a suitably defined domain. Discuss why the domain cannot be the set $\mathbb{R}$. Compare this curve with the curve of $y = x^2$, drawn on the same axes.

Ahmed does a parachute jump. He jumps out of the plane and falls faster towards the ground. After a few seconds his parachute opens. He slows down and then falls to the ground at a steady speed. Which of these graphs shows Ahmed's parachute jump? Explain why each of the other graphs is wrong.



Invent examples of functions with different definitions on different subdomains, for example, electricity charges as a function of the number of units of electricity used.

The Int *x function, written as* $[x]$*, maps x to the greatest integer less than or equal to x. Find* $[5.9]$*,* $[6]$ *and* $[-4.7]$.

Plot on the same axes the curves $y = 2^x$ *and* $y = 2^{-x}$ *for* $-3 \leq x \leq 3$*. Describe the features of the two curves. Discuss situations that could be modelled by these equations.*

A rectangular enclosure has a wall on one side and the other three sides are made of metal fencing. The side parallel to the wall has length d, measured in metres. The enclosure has an area of $600\,m^2$*. Show that the total length, L metres, of fencing is given by* $L = d + 1200/d$*. Plot this function using a graphics calculator. Find from the graph the value of d that makes L as small as possible.*

5.2 Recognise when a graph represents a functional relationship between two variables and when it does not.

Discuss whether or not the graphs of a circle and a semicircle represent functions. Identify any special cases.

5.3 Draw the tangent line at a point on the graph of a function, calculate the slope of this line and interpret the behaviour of the function at that point, knowing whether the function is increasing or decreasing at the point, or stationary.

Direct proportion

5.4 Translate the statement *y is proportional to* x^2 into the symbolism $y \propto x^2$ and into the equation $y = kx^2$; know that the graph of this equation is a parabola through the origin.

A body falling from rest under the force of gravity falls a distance s metres in time t seconds where $s = 4.9t^2$*. Find the distance fallen after 5 seconds. How long does it take the body to fall 30 metres?*

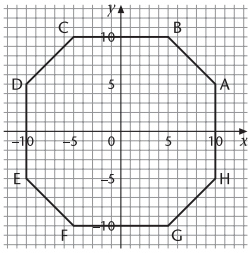
Discuss how to plot a linear graph $s = 4.9z$*, by defining the variable* $z = t^2$.

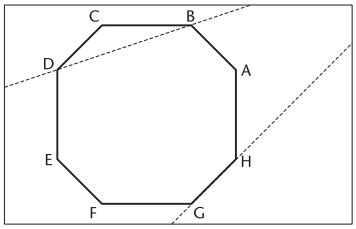
5.5 Recognise some other common examples of proportional variation.

Straight lines and linear functions

5.6 Know that a straight line in the explicit form $y = mx + c$ represents a function, but that a straight line in the implicit form $ax + by + d = 0$ may, or may not, be a function; know that any straight line in the xy-plane can be represented in this implicit form, but that only certain lines in the plane can be represented by the explicit form; work with both of these forms.

Look at this octagon.
The line through D and B has the equation $3y = x + 25$.
The line through G and H has the equation $x = y + 15$.





Solve the simultaneous equations

$3y = x + 25$

$x = y + 15$

to find the point of intersection of these two lines.

5.7 Plot the graphs of equations in 5.6 above; know the meanings of *gradient of the line* (and be familiar with alternative wordings such as *slope* or *rate of change of y with respect to x*) and *intercept on the x- or y-axis*, and relate these to the coefficients a, b and d, or to the coefficients m and c.

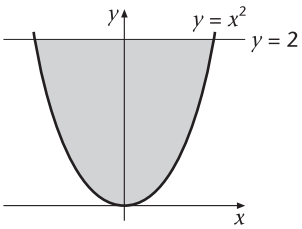
What is the gradient of the line $3x + 2y – 5 = 0$? Find an equation of a line that is perpendicular to this line. Draw the two lines on a graph.

A triangle has its vertices at the points $(1, 3)$, $(2, 5)$ and $(3, 4.5)$. Find the equations of the lines containing each side. Is the triangle a right-angled triangle? Explain how you know.

What angle does the line $y = \sqrt{3}x + 1$ make with the positive x-axis?

5.8 Graph regions of linear inequality and solve simple problems (e.g. elementary linear programming) represented by such regions; understand simple quadratic inequalities.

The shaded region is bounded by the curve $y = x^2$ and the line $y = 2$.
What two inequalities together fully describe the shaded region?



A company delivers new cars to Doha. It has a contract to deliver at least 65 cars each day. The company owns 7 carriers that can each carry 8 cars and 5 carriers that can each carry 10 cars. The company employs 8 drivers. Each carrier can make only one journey with a full load each day. What is the maximum numbers of cars that can be delivered each day? What is the minimum number of drivers needed to fulfil the contract?

Quadratic functions

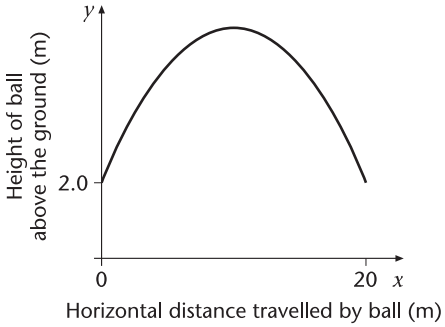
5.9 Recognise a second-order polynomial in one variable, $y = ax^2 + bx + c$, as a quadratic function; plot graphs of such functions (recognising that these are all parabolas), and pick out the intercepts with the coordinate axes, the axis of symmetry and the coordinates of the maximum or minimum point; understand when such functions are increasing, when they are decreasing and when they are stationary.

5.10 Model a range of situations with appropriate quadratic functions.

ICT opportunity
Include the use of a graphics calculator.

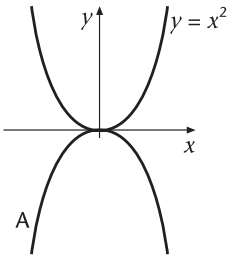
A fountain at ground level sprays out jets of water. Each jet is a parabola. The jet that sprays the farthest has equation $y = -x^2 + 8x - 15$. Factorise this expression.
Hence find: a. *where the fountain jet is positioned in this xy-coordinate system; and*
b. *how far from the fountain jet the water hits the ground. Calculate the greatest height that the water reaches.*

Huda throws a ball to Mariam who is standing 20 m away.
The ball is thrown and caught at a height of 2.0 m above the ground.



The ball follows the curve with equation $y = 6 + c(10 - x)^2$, where c is a constant. Calculate the value of c by substituting $x = 0$, $y = 2$ into the equation.

Curve A is the reflection in the x-axis of $y = x^2$. What is the equation of curve A?



An n-sided polygon has $\frac{1}{2}n(n-3)$ diagonals. How many diagonals has an octagon?
A polygon has 104 diagonals. How many sides does it have?

5.11 Solve quadratic equations exactly, by factorisation, by completing the square and by using the quadratic formula.

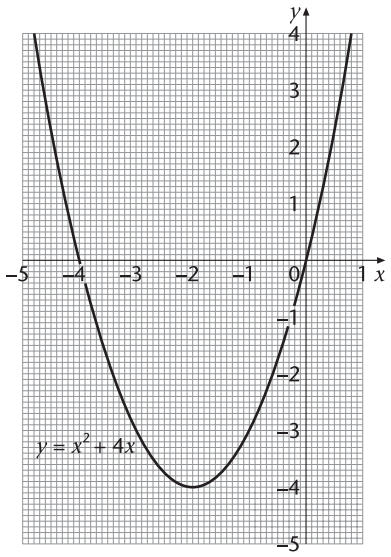
5.12 Find the axis of symmetry of the graph of a quadratic function and the coordinates of its turning point by algebraic manipulation; understand the effect of varying the coefficients a, b and c in the expression $ax^2 + bx + c$.

$y = (x - 3)^2 + 5$ is a quadratic function of x. What is the minimum value of this function and for what value of x does it occur? What is the maximum range of the function? Give the equation of the axis of symmetry of the function. Write an alternative form for the equation defining the function. Sketch the graph of this function.

5.13 Find approximate solutions of the quadratic equation $ax^2 + bx + c = 0$ by reading from the graph of $y = ax^2 + bx + c$ the x-coordinate(s) of the intersection point(s) of the graph of this function and the x-axis.

ICT opportunity
Include the use of a graphics calculator.

The graph below shows the curve $y = x^2 + 4x$.



a. *Solve the equation $x^2 + 4x - 2 = 0$ using the graph.*
Give your answers to two decimal places.

b. *The equation $x^2 + 4x + 5 = 0$ cannot be solved using the graph. Why not?*

5.14 Find exactly by algebraic means, and approximately from the points of intersection of a straight line with the graph of a quadratic function, the solution set of two simultaneous equations L_1 and Q_1, where L_1 represents a linear relation for y in terms of x, and Q_1 a quadratic function of y in terms of x.

ICT opportunity
Include the use of a graphics calculator.

5.15 Solve physical problems modelled simultaneously by two such functions.

Inverse proportion and the reciprocal function

5.16 Understand the statement *y is inversely proportional to x* and set up the corresponding equation $y = k/x$; know some characteristics, including that $x \neq 0$ and that $x = 0$ is an *asymptote* to the curve, as is $y = 0$; study examples of inverse proportionality.

ICT opportunity
Include the use of a graphics calculator.

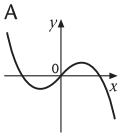
Three people working flat out complete a job in sixteen hours. How many hours would it take eight people to do the same job? Explain any assumptions you have made.

Look at the graphs below.

a. *One of the graphs shows the equation $y = kx - x^2$ (k is a constant).*
Which graph is it?

b. *One of the graphs shows the equation $y = k/x$, where k is a positive constant.*
Which graph is it?

A



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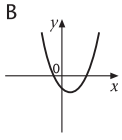

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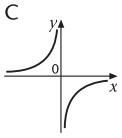

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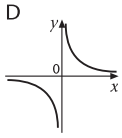

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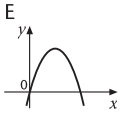

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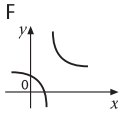


The average speed for a fixed distance journey is inversely proportional to the time taken to complete the journey. A family travels in Europe by car. They travel exactly half their journey in 2 hours, then stop for lunch for 1 hour, and then take 3 hours over the second half of the journey. How were the average speeds related on each part of the journey? If the average speed for the first half of the journey was 72 kilometres per hour what was the average speed for the whole journey?

Explain why the function $y = k/x$ cannot be defined on the domain set $\mathbb{R}$. What is the largest domain the function can be defined on? Sketch the graph of the function for this domain. Does the function have a greatest or least value? Is there anywhere where the function increases?

Geometry and measures

By the end of Grade 11, students continue to use their knowledge of geometry and trigonometry to solve practical and theoretical problems relating to shape and space. They solve right-angled triangles in two and three dimensions using the standard trigonometric ratios. They know and use the sine rule and cosine rule, and calculate the area of a triangle using $\frac{1}{2}ab\sin C$. They use Pythagoras' theorem to show that $\sin^2\theta + \cos^2\theta = 1$ for any angle θ, to find the distance between two points and to set up the Cartesian equation of a circle. They find the points of intersection of a straight line with a circle. They plot the graphs of circular functions and solve simple problems modelled by these functions. They prove standard circle theorems. They continue to use SI units and a range of measures to solve problems, including radian measure to calculate sector areas and arc lengths and compound measures. They use bearings, latitude, longitude and great circles to solve problems relating to position, distance and displacement on the Earth's surface. They use ICT to explore geometry.

Geometry and measures

Students should develop an appreciation of the importance and range of geometrical applications in the real world, and the aesthetic qualities of geometric models. They should understand the nature and place of geometric reasoning and proof, and how geometry may be related to algebraic concepts, and vice versa.

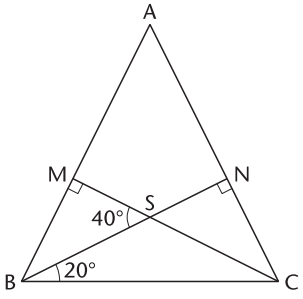
Students should:

6 Develop geometrical reasoning and proof and solve geometric problems

Congruence and similarity: properties of angles, straight lines and triangles

6.1 Use dynamic geometry systems to conjecture results and to explore geometric proof.

In the diagram below, the altitudes BN and CM of the triangle ABC intersect at S. ∠MSB is 40° and ∠SBC is 20°. Prove that triangle ABC is an isosceles triangle.



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Use of ICT

Geometry is enhanced with the use of dynamic geometry systems, or DGS, which provide an interactive means for investigating and hypothesising results which can then be proved as theorems.

Trigonometry, Pythagoras' theorem and the solution of triangles

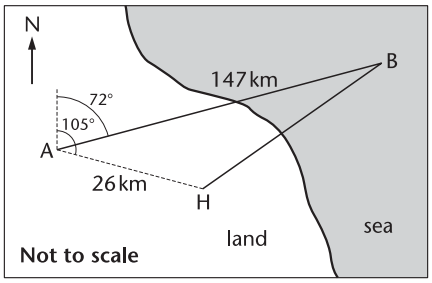
6.2 Solve right-angled triangles using the standard trigonometric ratios, including $\tan\theta = \sin\theta/\cos\theta$, and/or Pythagoras' theorem.

6.3 Know and use the sine rule and the cosine rule to solve triangles.

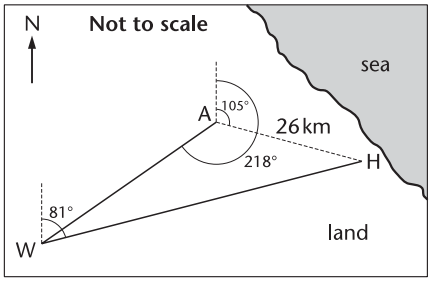
Show that Pythagoras' theorem is a special case of the cosine rule.

A ship sails 50 kilometres in a direction 032° and then 29 kilometres in a direction 315°. How far is the ship from its starting point? What is its bearing from its starting point?

A helicopter at airfield A received a distress call from a boat. The position of the boat, B, was given as 147 km from the airfield, on a bearing of 072°.
A man on the boat is flown to hospital. Calculate the distance the helicopter travelled from the boat to the hospital at H.



On another occasion the helicopter travelled from the airfield on a bearing of 218° to fly a pregnant woman at W to the hospital. The helicopter then flew on a bearing of 081° to the hospital, H. Calculate the distance the helicopter travelled from where it picked up the woman to the hospital.



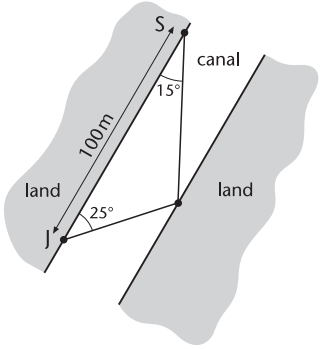
A triangle has its three angles in the ratio 2 : 3 : 4. Find to two significant figures the ratio of the lengths of its sides.

6.4 Solve triangle problems in two and three dimensions.

Triangle problems

Include the terminology *angle of inclination* and *angle of declination*.

The two sides of a canal are straight, parallel and the same height above the water level. Jana and Shrifa want to find the width of the canal. They measure 100 m on the canal bank and stand facing each other at the points J and S. Jana measures the angle she turns through to look at the post, P, as 25°. Shrifa measures the angle she turns through to look at the post as 15°. Calculate the width of the canal.



Not to scale

The Great Pyramid of Cheops in Egypt is built on a square base with side 230 metres. Each face of the pyramid is at 52° to the horizontal. Calculate the height of the pyramid. Calculate the inclination of an edge of the pyramid to the horizontal.

The Great Pyramid of Cheops at Giza. Source: www:kingtutshop.com

6.5 Calculate the area of a triangle using $\frac{1}{2}ab\sin C$.

6.6 Use Pythagoras' theorem to find the distance between two points in the Cartesian plane; set up the Cartesian equation of a circle of radius r, centred at the point (α, β).

Find the equation of a circle of radius 5 units, centred at the point (5, –3).

Find the exact distance between the point (1, 4) *and the point* (–2, 5).

Two sides of a right-angled triangle are of length 21 cm and 29 cm. What are the possible lengths of the remaining side?

Pythagoras' theorem

Many interesting websites are devoted to proofs and applications of this key theorem in geometry.

6.7 Find the points of intersection of a straight line with a circle by using algebraic substitution from the equation of the straight line into the equation of the circle.

Find the points where the line $4x - 3y = 0$ *cuts the circle* $x^2 + y^2 = 100$.

Circular functions

6.8 Use the unit circle $x^2 + y^2 = 1$ to plot graphs of the circular functions $\sin\theta°$ and $\cos\theta°$ for any angle $\theta°$, where $0° \le \theta° \le 360°$; know that any point on this circle has coordinates $(\cos\theta°, \sin\theta°)$, where $\theta°$ is the angle the radius to the point makes with the positive x-axis.

Explain why $\sin(180° - \theta°) = \sin\theta°$.

Give the exact value of $\cos 225°$. *What other angle has the same cosine value?*

6.9 Derive and recall the exact values for the sine, cosine and tangents of 0°, 30°, 45°, 60°, 90° and use these in relevant calculations.

Calculate the exact area of an equilateral triangle with sides of length 6 cm.

6.10 Use a calculator to find sine and cosine values of a given angle and to find the angle corresponding to a given value of the sine or cosine of that angle, and know that these are inverse functions defined on a restricted domain.

Find the angle whose sine is 0.9063. What is the cosine of this angle? For this angle, verify the result in GM 6.11.

6.11 Use Pythagoras' theorem to show that $\sin^2\theta° + \cos^2\theta° = 1$ for any angle $\theta°$.

Verify this result for the angles 30°, 45° and 60°. What happens when $\theta° = 90°$?

Powers of (co)sines

Note that $(\cos\theta°)^2$ is written as $\cos^2\theta°$ and that $(\sin\theta°)^2$ is written as $\sin^2\theta°$.

6.12 Solve simple problems modelled by circular functions.

Radian measure and circle geometry

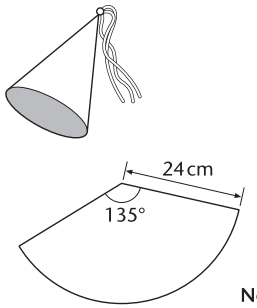
6.13 Use radian measure to calculate sector areas and arc lengths.

Sectors and arcs

Include terms associated with a circle: *centre, radius, diameter, circumference, arc length, sector, segment, chord*.

A satellite is 1500 km above the Earth. It has a camera with a 50° angle of view with which it surveys the Earth below. Draw a diagram to represent the satellite and its camera in relation to the Earth. Calculate how far apart the two furthest points on the Earth are that can be photographed by the satellite at any one time. Take the Earth to be a sphere of radius 6378 kilometres.

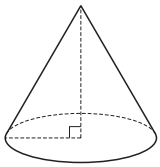
A manufacturer makes party hats shaped like hollow cones. To make the hats she cuts pieces of card that are sectors of a circle, radius 24 cm. The angle of the sector is 135°.



Not to scale

a. *Show that the arc length of the sector is 18π cm.*

b. *The sector is joined edge to edge to make a cone. The edges of the sector meet exactly with no overlap. Calculate the vertical height of the completed hat.*



6.14 Prove and use the theorems:

- The perpendicular from the centre of a circle to a chord bisects the chord.
- The two tangents from an external point to a circle are of equal length.
- The angle subtended by an arc at the centre of the circle is twice the angle subtended by the arc at a point on the circle, including, as a special case, the angle in semicircle is a right angle.
- Angles in the same segment subtended by a chord are equal.
- The angle subtended by a chord at the centre of a circle is twice the angle between the chord and the tangent to the circle at an end point of the chord.
- When two chords BC and DE in a circle intersect at A then $AB \times AC = AD \times DE$.
- Opposite angles of a cyclic quadrilateral are supplementary.

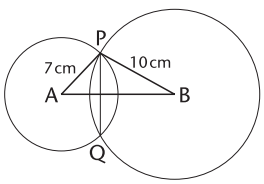
Circles

Include terms associated with a circle: *centre, radius, diameter, circumference, arc length, sector, segment, chord, tangent, inscribed circle, circumcircle, cyclic quadrilateral*.

Circle theorems

Include the use of dynamic geometry systems (DGS).

Two circles with centres at A and B have radii of 7 cm and 10 cm as shown in the diagram. The length of the common chord PQ is 8 cm. Calculate the length of AB.



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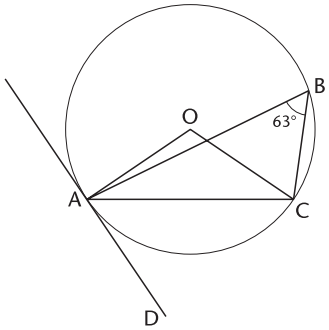
In the diagram below, AD is a tangent to the circle with centre O.
∠ABC is 63° and AC is a chord of the circle. AB is 18 cm and BC is 3 cm.

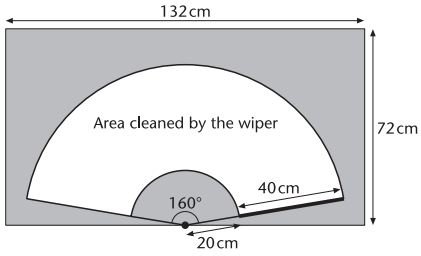


Calculate the values of ∠AOC, ∠OCA and ∠CAD.
Calculate the area of triangle ABC.
Calculate the length of AC.

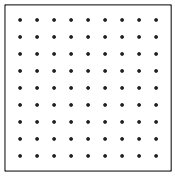
7 Use a range of measures and compound measures to solve problems

7.1 Calculate lengths, areas and volumes of geometrical shapes.

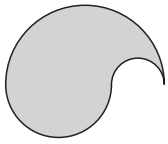
The diagram models a rectangular rear windscreen of a car. The windscreen wiper can rotate through 160°. Calculate the percentage of the rear window that is cleaned by the wiper.



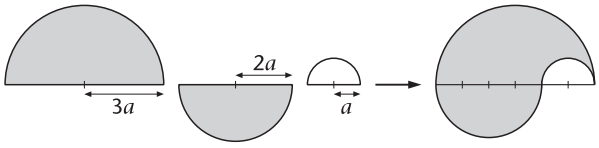
On the pinboard, draw a trapezium that has a perimeter of $6 + 4\sqrt{2}$.



This shape is designed using three semicircles.



The radii of the semicircles are 3a, 2a and a.



a. *Find the area of each semicircle, in terms of a and π, and show that the total area of the shape is $6\pi a^2$.*

b. *Find a when the area is 12 cm^2, leaving your answer in terms of π.*

7.2 Use bearings, latitude, longitude and great circles to solve problems relating to position, distance and displacement on the Earth's surface.

How would you find the shortest distance between Doha and London?

A plane flies from Doha to Karachi almost along the line of latitude 25 degrees north. Doha is at longitude 51 degrees east approximately and Karachi is at longitude 67 degrees east approximately. How far is it from Doha to Karachi along this route?

What is a great circle of the Earth?

An oil tanker sails 350 km from Doha towards Dubai on a bearing of 090° and then from Dubai towards Al Kuwayt on a bearing 310°. Al Kuwayt is about 600 km from Doha. Approximately, how far is it from Dubai to Al Kuwayt?

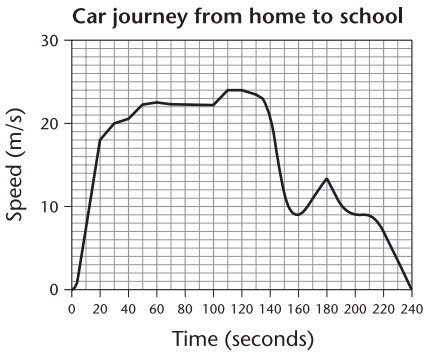
7.3 Work with SI units and compound measures including density, average speed and acceleration, measures of rate, and population density (number of people per unit area), using appropriate units and dimensions.

Compound measures

Use appropriate SI units and dimensions. Stress how units are calculated in compound measures.

Link where appropriate to work in science and technology, using compound measures such as rate of doing work or momentum.

Wafa recorded the speed of a car every 10 seconds throughout a short journey from her home to school. She used the data to draw a speed–time graph.

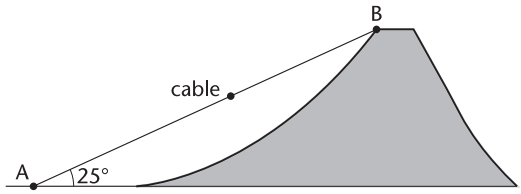


a. *Find a point during the journey when the car's speed was increasing most quickly. Mark this point as P. By drawing appropriate lines on the graph, find the acceleration of the car at point P.*

b. *Find a point during the journey when the car's speed was decreasing most quickly. Mark this point as Q. By drawing appropriate lines on the graph, find the acceleration of the car at point Q.*

c. *The car uses least fuel when it travels at speeds between 20 m/s and 25 m/s. Find an approximate value for the area under the graph for the period when the car was travelling at between 20 m/s and 25 m/s. What does this area represent? Give the correct units.*

A cable car takes passengers to the top of a volcano. It starts from station A and takes 16 minutes to reach station B at the top of the volcano. The average speed of the cable car is 2 metres per second. The cable car is at an angle 25° to the horizontal. Find, to the nearest metre, the height of the volcano as measured from A.



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Probability and statistics

By the end of Grade 11, students plan questionnaires and surveys to collect meaningful primary data from samples. They know the importance of representative samples, and can locate sources of bias. They collect data from secondary sources, including the Internet, and ask and answer questions related to the data. They group data and plot histograms and other frequency and relative frequency distributions. They draw stem-and-leaf diagrams and box-and-whisker plots, and continue to calculate and use measures of central tendency. They analyse results to draw conclusions and use a range of graphs, charts and tables to present their findings.

Probability and statistics

Students should know that statistics is the branch of mathematics used to predict the outcomes of large numbers of events when these outcomes are uncertain, and that probability lies at the heart of statistics. They should be aware of the uses of statistics in society and recognise when statistics are used sensibly and when they are misused or likely to be misunderstood.

Students should:

8 Collect, process, represent, analyse and interpret data and reach conclusions

Sampling

8.1 Know that:

- it is important to choose representative samples;
- in a random sample there are chance variations;
- in a biased sample there are systematic differences between the sample and the population from which it is drawn;

and locate obvious sources of bias within a sample.

An article in a newspaper claimed that 93% of us drop litter every day. It does not say how the journalist knows that 93% of people drop litter every day. Some students think the percentage of people who drop litter every day is much lower than 93%. They decide to do a survey.

a. *Jabor plans to ask 10 people if they drop litter every day. Give two different reasons why Jabor's method might not give very good data.*

b. *Layla plans to go into Doha on Sunday morning. She will stand outside a shop and record how many people walk past and how many of those drop litter. Give two different reasons why Layla's method might not give very good data.*

Mosa wants to investigate whether more people are born in the winter than in the summer. He plans to ask 30 students in his grade whether they were born in the winter or the summer. Discuss ways in which Mosa could improve his survey.

Introductory statistical techniques

8.2 Plan surveys and design questionnaires to collect meaningful primary data from samples in order to test hypotheses about or estimate characteristics of the population as a whole; formulate problems using secondary data from published sources, including the Internet.

Statistical techniques

These lay the foundations for later development in the advanced mathematics course on quantitative methods.

8.3 Calculate and use measures of central tendency such as the arithmetic mean and the median.

Investigate life expectancy in a range of countries including Qatar, Iran, Turkey, India, Brazil, China, Russia, Italy, the United Kingdom and the United States of America.

8.4 Construct (relative frequency) histograms and plot cumulative frequency distributions, grouping continuous data when necessary.

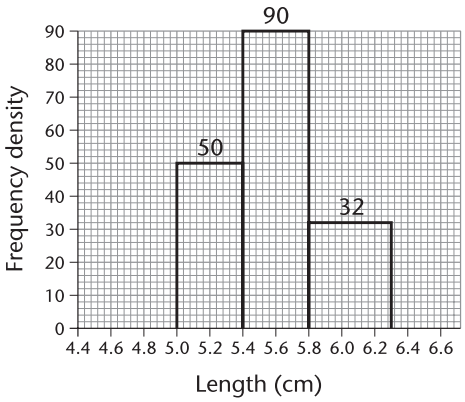
Histograms and cumulative frequency distributions

Include the terms *frequency*, *frequency distribution*, *frequency density*, *relative frequency* and *relative frequency distribution*, and *range, percentile, interquartile range, semi-interquartile range*, and *mode, modal class, modal frequency*.

A scientist wanted to investigate the lengths of an egg from a particular breed of hen. Taking a sample of 80 eggs, she measured the length of each one and grouped the data as follows:

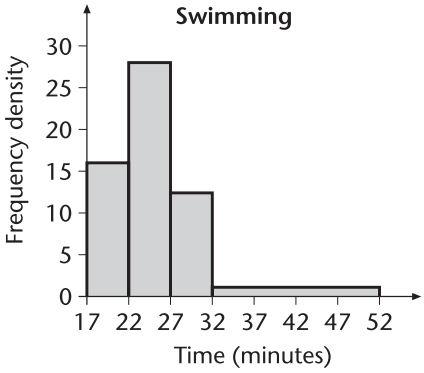
Length (l) in cm	$4.4 \le l < 5.0$	$5.0 \le l < 5.4$	$5.4 \le l < 5.8$	$5.8 \le l < 6.3$	$6.3 \le l < 6.5$
Frequency	4	20	36	16	4

Complete the histogram to show this information. Write the frequency density on each part of the histogram.



Calculate the mean length of the eggs in her sample.
Discuss how to calculate best estimates for the modal value and the median value of the lengths of the eggs in the sample.

304 people took part in a swimming contest. They swam 1.5 km. The histogram shows the distribution of their times for the event.



a. *The histogram is constructed using frequency densities.*

The table shows the frequency densities. Complete the table to show the frequencies.

Time t (minutes)	Frequency density	Frequency
$17 \le t < 22$	16.0	80
$22 \le t < 27$	28.0	
$27 \le t < 32$	12.4	
$32 \le t < 52$	1.1	

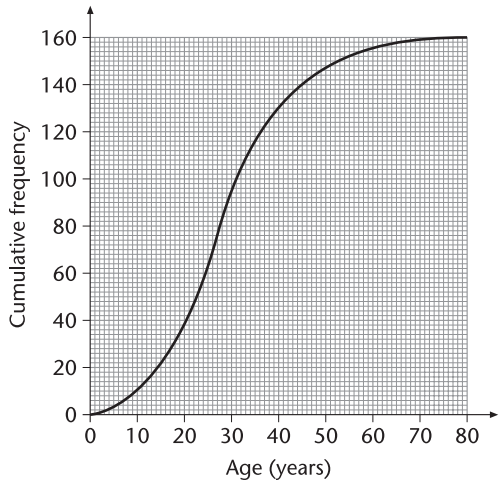
b. *304 people took part. Calculate an estimate of the mean time for swimming.*

c. *Explain why the median time for swimming must be between 22 and 27 minutes.*

d. *Calculate an estimate of the median time for swimming.*

Sulaiman did a survey of the age distribution of 160 people at a theme park.

The cumulative frequency graph shows his results.



a. *Use the graph to estimate the median age of the 160 people at the theme park.*

b. *Use the graph to estimate the interquartile range of the age of the 160 people at the theme park.*

8.5 Draw stem-and-leaf diagrams and box-and-whisker plots and use them in presenting conclusions.

Stem plots and box plots

Stem-and-leaf diagrams and box-and-whisker plots are also known as stem plots and box plots.

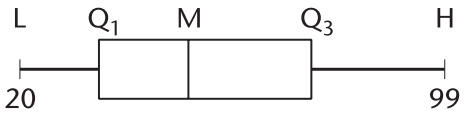
A hospital clinic records the number of patients seen each day in a stem-and-leaf diagram.

12	5	6	7	8	9			
13	0	1	2	2	4	7	8	9
14	0	1	2	3	5	9		

Key: 12 | 5 means 125 patients.

Find the range, the median and the mode.

The diagram shows a box-and-whisker plot of examination marks for a class of students.



L represents the lowest mark scored and H is the highest mark scored. LH then represents the range of marks. Q_1 is the first quartile mark, Q_3 is the third quartile mark and Q_1Q_3 is the interquartile range. M is the median mark.

A school for boys and a school for girls each enters students for the same mathematics examination. The girls' marks were:

97 98 57 45 63 75 87 34 56 28 67 89 45 61 53 49 81 32 23 45 47 72 34 54 23 100 76 47.

The boys' marks were:

67 87 83 92 34 31 23 25 29 39 89 91 54 47 41 50 77 18 89 10 26 62 39 14 90.

Draw back-to-back stem-and-leaf diagrams to represent these scores. Compare the performance of the girls and the boys, explaining your methodology and findings.

Using the above data, plot a cumulative frequency graph for the marks of the girls.
What was the median score?
What was the interquartile range of the distribution of marks?
Draw a box-and-whisker plot to represent the girls' marks.

Draw a relative frequency histogram for these data, explaining how the data were grouped and the meaning of each bar of the histogram.

8.6 Make inferences and draw conclusions from the formulation of a problem to the collection and analysis of data in a range of situations; use a range of statistics and graphs, charts and tables to present and justify findings.

9 Use of ICT

9.1 Use a calculator with statistical functions to aid the analysis of large data sets, and ICT applications to present statistical tables and graphs.

ICT opportunity

A range of ICT applications can support data handling.