Mathematics standards

Grade 12
Foundation

Summary of students' performance by the end of Grade 12

Reasoning and problem solving

Students solve routine and non-routine problems in a range of mathematical and other contexts and use mathematics to model and predict the outcomes of substantial real-world applications. They identify and use connections between mathematical topics. They break down complex problems into smaller tasks, and set up and perform appropriate manipulations and calculations. They develop and explain longer chains of logical reasoning, using correct mathematical notation and terms, and generate mathematical proofs. They aim to generalise. They approach problems systematically, knowing when it is important to enumerate all outcomes. They conjecture alternative possibilities with 'What if …?' and 'What if not …?' questions. They synthesise, present, interpret and criticise mathematical information, working to expected degrees of accuracy. They recognise when to use ICT and do so efficiently.

Number and algebra

Students appreciate a wide range of numerical and algebraic applications in the real world. They rearrange harder formulae connecting two or more variables and generate further formulae from physical contexts. They generate recursive sequences to model the behaviour of real-world situations. They use physical contexts to plot and interpret the graphs of linear, quadratic, cubic, reciprocal, exponential and logarithm functions, the sine and cosine functions, the modulus function and other simple non-standard functions. They solve a range of problems using inverse and composite functions. They apply combinations of transformations to the graph of the function $y = \mathrm{f}(x)$. They use realistic data and ICT to analyse problems.

Geometry and measures

Students use approximation methods to calculate the area of an irregular two-dimensional flat surface and the volume of a prism with a constant, but irregular-shaped, cross-section. They draw and use plans and elevations, and interpret maps and scale drawings. They translate, reflect, rotate and enlarge two-dimensional geometric objects. They begin to use vectors to solve physical problems. They solve a range of problems involving compound measures, using appropriate units and dimensions. They explore aspects of geometry using ICT.

Probability and statistics

Students arrive at conclusions from the formulation of a problem to the collection and analysis of data in a range of situations. They use secondary data from published sources, including the Internet. They use ICT to calculate statistical quantities and to produce a range of graphs, charts and tables to present and justify their findings. They calculate measures of spread, including the variance and standard deviation. They construct histograms and plot cumulative frequency distributions, using grouped continuous data if necessary. They understand that a random variable has a range of values that cannot be predicted with certainty and investigate common examples. They measure the empirical probability (relative frequency) of obtaining a particular value of a random variable. They use a simple mathematical model to calculate the theoretical probability of obtaining a particular outcome for a random variable associated with a set of events. They calculate probabilities of single and combined events, and understand risk as the probability of the occurrence of an adverse event. They use tree diagrams to represent and calculate the probabilities of compound events when the events are independent and when one event is conditional on another. They use simple simulations and consider trends over time using a moving average.

Content and assessment weightings for Grade 12

The foundation mathematics standards are grouped into four strands: reasoning and problem solving; number and algebra; geometry and measures; and probability and statistics.

The reasoning and problem solving strand cuts across the other three strands. Reasoning, generalisation and problem solving should be an integral part of the teaching and learning of mathematics in all lessons.

The weightings of the content strands relative to each other are as follows:

Foundation	Number and algebra	Geometry and measures*	Probability and statistics
Grade 10	55%	30%	15%
Grade 11	55%	30%	15%
Grade 12	50%	25%	25%

* including trigonometry

The standards are numbered for easy reference. Those in shaded rectangles, e.g. 1.2, are the performance standards for all foundation students. The national tests for foundation mathematics will be based on these standards.

Grade 12 teachers should review and consolidate Grades 10 and 11 standards where necessary.

Mathematics standards

Grade 12
Foundation

Reasoning and problem solving

By the end of Grade 12, students solve routine and non-routine problems in a range of mathematical and other contexts, and use mathematics to model and predict the outcomes of substantial real-world applications. They identify and use connections between mathematical topics. They break down complex problems into smaller tasks, and set up and perform appropriate manipulations and calculations. They develop and explain longer chains of logical reasoning, using correct mathematical notation and terms, and generate mathematical proofs. They aim to generalise. They approach problems systematically, knowing when it is important to enumerate all outcomes. They conjecture alternative possibilities with 'What if …?' and 'What if not …?' questions. They synthesise, present, interpret and criticise mathematical information, working to expected degrees of accuracy. They recognise when to use ICT and do so efficiently.

Students should:

1 Use mathematical reasoning to solve problems

1.1 Solve routine and non-routine problems in a range of mathematical and other contexts, including open-ended and closed problems.

1.2 Use mathematics to model and predict the outcomes of substantial real-world applications; compare and contrast two or more given models of a particular situation.

1.3 Identify and use interconnections between mathematical topics.

1.4 Break down complex problems into smaller tasks.

1.5 Use a range of strategies to solve problems, including working the problem backwards and then redirecting the logic forwards; set up and solve relevant equations and perform appropriate calculations and manipulations; change the viewpoint or mathematical representation, and introduce numerical, algebraic, graphical, geometrical or statistical reasoning as necessary.

1.6 Develop longer chains of logical reasoning, using correct mathematical notation and terms.

1.7 Explain their reasoning, both orally and in writing.

1.8 Generate simple mathematical proofs, and identify exceptional cases.

1.9 Generalise when appropriate.

1.10 Approach a problem systematically, recognising when it is important to enumerate all outcomes.

1.11 Conjecture alternative possibilities with 'What if …?' and 'What if not …?' questions.

Key standards

Key performance standards are shown in shaded rectangles, e.g. 1.2.

Cross-references

Standards are referred to using the notation RP for reasoning and problem solving, NA for number and algebra, GM for geometry and measures, and PS for probability and statistics, e.g. standard NA 2.3.

Examples of problems

The examples of problems in italics are intended to clarify the standards, not to represent the full range of possible problems.

Reasoning and problem solving

Reasoning, generalisation and problem solving should be an integral part of the teaching and learning of mathematics in all lessons.

Proofs

Relate to the mathematics in the other strands.

1.12 Synthesise, present, discuss, interpret and criticise mathematical information presented in various mathematical forms.

1.13 Work to expected degrees of accuracy, and know when an exact solution is appropriate.

1.14 Recognise when to use ICT and when not to, and use it efficiently.

Number and algebra

By the end of Grade 12, students appreciate a wide range of numerical and algebraic applications in the real world. They rearrange harder formulae connecting two or more variables and generate further formulae from physical contexts. They generate recursive sequences to model the behaviour of real-world situations. They use physical contexts to plot and interpret the graphs of linear, quadratic, cubic, reciprocal, exponential and logarithm functions, the sine and cosine functions, the modulus function and other simple non-standard functions. They solve a range of problems using inverse and composite functions. They apply combinations of transformations to the graph of the function $y = \text{f}(x)$. They use realistic data and ICT to analyse problems.

Algebra

Students should learn that algebra enables generalisation and the establishment of relationships between quantities and/or concepts. They should understand the nature and place of algebraic reasoning and proof, and how algebra may be related to geometric concepts, and vice versa.

Students should:

2 Identify and use number sets

2.1 Make appropriate use of their knowledge of number sets from Grades 10 and 11.

3 Use index notation and solve numerical problems

3.1 Develop further confidence in all the calculation skills established in Grades 10 and 11.

Farida is making a scale model of the Earth and the Moon for a museum. She has found out the diameters of the Earth and the Moon, and the distance between them in metres.

Diameter of the Earth	1.28×10^7 m
Diameter of the Moon	3.48×10^6 m
Distance between Earth and Moon	3.89×10^8 m

a. *How many times bigger is the diameter of the Earth than the diameter of the Moon?*

b. *In Farida's scale model the diameter of the Earth is 50 cm. What should be the distance between the Earth and the Moon in Farida's model?*

Look at the table.

	Earth	Mercury
Mass (kg)	5.98×10^{24}	3.59×10^{23}
Atmospheric pressure (N/m^2)		2×10^{-8}

The atmospheric pressure on Earth is 5.05×10^{12} times as great as the atmospheric pressure on Mercury. Calculate the atmospheric pressure on Earth.

4 Generate and manipulate algebraic expressions and formulae, and solve algebraic equations

4.1 Rearrange harder formulae connecting two or more variables and generate further formulae from physical contexts.

The mathematician Johannes Kepler set out three laws of planetary motion in his famous book 'The Harmony of the World', published in 1619. Kepler's third law of planetary motion states that the square of the period of revolution of a planet about the Sun is proportional to the cube of the mean distance of the planet from the Sun. Write this statement as a mathematical equation.

The value of a new car depreciates by 20 per cent at the end of the first year and then loses value at the rate of 10 per cent for every subsequent year. Set up a formula to describe the value V of the car t years after purchase. After how many years will the car be worth one quarter of its purchase price?

4.2 Generate recursive sequences from term-to-term and position-to-term definitions to model the behaviour of real-world situations, for example population growth.

In a certain country, there is a net increase in population from one year to the next of 5 per cent. Set up a recurrence relation to describe the population in year n + 1 in terms of the population in year n. Find the population in year n + 4 compared to the population in year n. Use your formula to find the number of years it takes to double the population from year n.

A woman buys a car and pays in monthly instalments. The car costs QR 60 500 and interest is charged on any outstanding debt at a monthly rate of r%. The woman pays back a fixed amount each month of QR M. Set up a recurrence relation connecting the amount owed, A_{n+1}, after n + 1 months in terms of the amount owed, A_n, at the end of the nth month. How many months will it take to repay the debt if M = QR 1200 and r = 1.2%? How much will the woman have then paid for the car? Investigate repayments for different values of M and r.

ICT opportunity

Use spreadsheets in examples like these.

5 Generate and solve problems with functions and graphs

Functions and their inverses

5.1 Use a graphics calculator, including the trace function, to show approximate solutions to physical problems requiring the location and physical interpretation of the intersection points of two or more graphs.

5.2 Use physical contexts to plot and interpret:

- graphs of linear, quadratic and cubic functions;
- graphs of the reciprocal function $y = k/x$ $(x \neq 0)$;
- graphs of the sine and cosine functions;
- graphs of the modulus function and a range of simple non-standard functions.

Which grows faster for $x \geq 0$: the power function $y = x^3$ or the exponential function $y = e^x$? Justify your answer. (See also NA 5.8.)

Investigate physical examples of inverse square laws.

Find physical examples that are modelled by circular functions.

A big wheel makes one complete revolution every 90 seconds. The wheel has a diameter of 20 metres. The bottom of the wheel is 2 metres above the ground. Two people get on the wheel and sit in a seat, and then the wheel starts to rotate. T seconds later their height above the ground is given by $h = 2 + 8\sin 4T°$. Explain why this is an appropriate formula to use. At what two consecutive times are they 12 m above the ground?

Modelling with functions and their inverses

This aspect of mathematics adds realism, shows the importance of the subject through application and motivates students. Where possible, the use of real data and its analysis through ICT should be encouraged.

Modelling with circular functions

Examples could include oscillations on a spring, bungee jumping, pulse rate, blood pressure, alternating currents, daylight hours.

A ship can only enter a harbour when the tide is in; it must have a minimum depth of water of 8 metres. The tide follows a daily sinusoidal variation given by the formula $d = 5 \sin 15t° + 8$, *where t is the time in hours from midnight onwards, measured on the 24-hour clock. At how many times in a day will the depth of water in the harbour be exactly 8 m? For how many hours a day can the ship enter the harbour? Sketch how the level of the tide varies with the time of the day.*

5.3 Find, graph and use the inverse function of those functions in NA 5.2 given by a one-to-one mapping or restricted to such mappings; know that the graph of the inverse function may be found by reflecting the graph of the function in the line $y = x$; solve a range of problems using inverse functions.

The cost of production of q silver bracelets is $C = 200 + 15q$.
Find the inverse function and interpret its meaning.

5.4 Add, subtract and multiply two functions given in the form $y_1 = f_1(x)$ and $y_2 = f_2(x)$; write down, without simplification, the mathematical form for one function divided by another.

5.5 Understand the concept of a composite function and use the notation $y = f(g(x))$.

5.6 Deconstruct a composite function into its constituent functions, using inverse functions.

Calculate the inverse function of $f(x) = 5x - 8$.

Starting from the function $y = x$, *describe how the function* $y = (5x - 3)^2$ *is constructed. Show how to deconstruct this function back to the original function.*

Composite functions

Use a 'function machine' to introduce the idea of a composite function and its inverse.

Transformation of functions

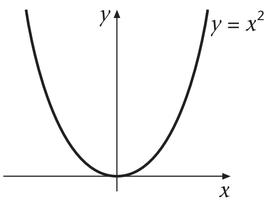
5.7 Understand the transformations of the function $y = f(x)$ given by:

- $y = f(x) + a$, representing a translation by a in the positive y-direction;
- $y = f(x - a)$, representing a translation by a in the positive x-direction;
- $y = af(x)$, representing a stretch with scale factor a parallel to the y-axis;
- $y = f(ax)$, representing a stretch with scale factor $1/a$ parallel to the x-axis;

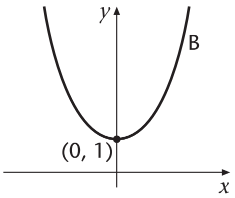
use these and combinations of these transformations to sketch, stage by stage, the transformation of the graph of $y = f(x)$ into the graph of the transformed function.

The straight line $y = mx + c$ *is the straight line* $y = mx$ *translated parallel to itself a distance c in the y-direction. When* $y = mx$, *the variable y is directly proportional to the variable x. By redefining the origin to the point* $(0, c)$ *the straight line* $y = mx + c$ *implies that the variable* $(y - c)$ *is directly proportional to the variable x, since* $y - c = mx$ *and this passes through the point* $(0, c)$.

The diagram shows the graph with equation $y = x^2$. *On the same axes, sketch the graph with equation* $y = 2x^2$.



Curve B is the translation, one unit up the y-axis, of $y = x^2$. *What is the equation of curve B? Translate curve B two units to the left. What is the equation of this new curve?*



A function is defined by f(x) = x^2. *Describe the functions* a. f(x – 2) *and* b. f(x + 1), *stating how the graphs of each function relate to the graph of* y = f(x), *and give the defining equation for each function.*

Transform the curve $y = x^3$ *into the curve* $y = 5x^3$. *Describe the effect of the transformation. The curve is then translated one unit in the positive x-direction. What is the equation of this new curve?*

Describe in words how the graph of $y = 1/x$ *is transformed into the graph* $y = 4 + 5/x$. *Sketch each graph on the same set of axes.*

Explain the difference between a. *the functions* $y = \cos x°$ *and* $y = \cos(x + 45)°$ *and* b. *the functions* $y = \cos x°$ *and* $y = 2\cos x°$.

Modelling with exponential functions

5.8 Understand the ideas of exponential growth and decay and the forms of the associated graphs $y = a^x$, where $a > 0$; use a graphics calculator to plot the graphs of the exponential function, e^x, and the natural logarithm function, $\ln x$; know that one is the inverse function of the other and use this to find solutions to physical problems; solve for x the equation $y = a^x$ and use this in problems; use the log function (logarithm in base 10) on a calculator.

The growth of the Internet since 1990 has been modelled by the function $N = 0.2(1.8)^t$, *where N is the number of users, counted in millions, t years from 1990. Plot the graph of this function. How many Internet users does the model predict for the year 2006?*

When living organisms die the amount of carbon-14 present in the dead matter decays exponentially according to the formula $N = N_0 e^{-0.000121t}$, *where* N_0 *is the initial quantity and t is the time in years. A bone uncovered at an archaeological site has 35% of its original carbon-14. Estimate the age of the bone. After how many more years will the bone have only 25% of its carbon-14?*

The number of bacteria in a colony of bacteria grows exponentially. At 1300 hours yesterday the number of bacteria was 1000 and at 1500 hours it was 4000. How many bacteria were there at 1800 hours yesterday? How many bacteria will there be at 1000 hours today?

The Global Report estimated the population of the world in 1975 as 4.1 billion people and that it was growing at the rate of 2% per year. Set up an equation to predict the world population t years from 1975. Use this model to predict the world's population in 2020. Discuss any assumptions you have made.

Earthquakes produce oscillations in the ground. The strength, S, of the quake is measured on the Richter scale and is given by $S = \log A$, *where A is the measured amplitude of the oscillation as measured in millimetres on a calibrated seismograph. What amplitude of oscillation corresponds to a major earthquake with a Richter scale value of 7.8? What is the Richter scale value of an earthquake with an oscillation that has an amplitude of 2000 mm?*

Modelling with exponential functions

Include further examples to illustrate population growth and decay, radioactivity, cooling, drug absorption, spread of an epidemic, and compound interest.

Geometry and measures

By the end of Grade 12, students use approximation methods to calculate the area of an irregular two-dimensional flat surface and the volume of a prism with a constant, but irregular-shaped, cross-section. They draw and use plans and elevations, and interpret maps and scale drawings. They translate, reflect, rotate and enlarge two-dimensional geometric objects. They begin to use vectors to solve physical problems. They solve a range of problems involving compound measures, using appropriate units and dimensions. They explore aspects of geometry using ICT.

Geometry and measures

Students should develop an appreciation of the importance and range of geometrical applications in the real world, and the aesthetic qualities of geometric models. They should understand the nature and place of geometric reasoning and proof, and how geometry may be related to algebraic concepts, and vice versa.

Students should:

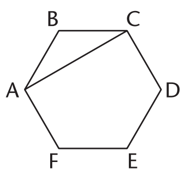
6 Develop geometrical reasoning and proof, and solve geometric problems

Congruence and similarity: properties of angles, straight lines, triangles and circles

6.1 Use ICT to investigate a range of geometrical situations, including:

- the generation of geometric patterns, including Islamic patterns;
- similarity and congruence;
- constructions;
- plans and elevations.

Each side of the regular hexagon ABCDEF is 10 cm long.
Find the length of the diagonal AC.



TIMSS Grade 12

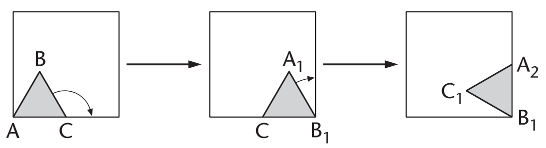
Use of ICT

Geometry is enhanced with the use of a dynamic geometry system, or DGS, which provides an interactive means for investigating and hypothesising results in geometrical situations.

7 Work with transformations and projections

7.1 Transform rectilinear figures using combinations of translations, rotations about a centre of rotation, enlargements about a centre of enlargement, and reflections about a line; understand the meanings of positive, negative and fractional scale factors in enlargements.

An equilateral triangle ABC has side length 10 cm. It rotates around the inside of a square of side length 20 cm.



a. *Triangle ABC rotates about C to the position shown as CA_1B_1. What is the angle of rotation?*

b. *Calculate the distance along the path travelled by point A in turning from A to A_1.*

c. *Calculate the distance along the path travelled by point A in turning from A_1 to A_2.*

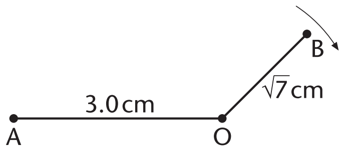
Transformations

Transformations are best developed through use of DGS.

d. *The triangle continues rotating around the inside of the square in the same way until it is back at the original position. Which of the original points A, B or C will point A land on when it has completed its rotations around the inside of the square?*

A triangle has vertices at the points (4, 5), (6, 1) *and* (8, 11). *The triangle is enlarged by a factor of 2 about a centre of enlargement at the point* (3, –3). *Draw the enlarged triangle in its correct position on a coordinate grid.*

The line segment OA is 3.0 cm long. The line segment OB is $\sqrt{7}$ *cm long. OB can rotate in a horizontal plane about the point O.*



a. *Find the maximum possible distance B can be from A. Explain whether your answer is a rational number or an irrational number.*

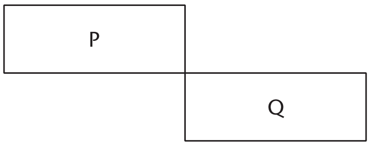
b. *Find the minimum possible distance B can be from A. Explain whether your answer is a rational number or an irrational number.*

c. *Sketch a different position for the line segment OB so that the distance from A to B, AB, is a rational number. Confirm by calculation that your answer is a rational number.*

d. *OB is reduced in length to 2.6 cm. OA is still 3.0 cm long. Calculate the distance AB when angle AOB is 120°.*

e. *The lengths of 2.6 cm and 3.0 cm are accurate to one decimal place. The 120° angle is accurate to the nearest degree. Calculate the greatest and least possible values of AB.*

The diagram shows two rectangles, P and Q.



The rectangle Q in the diagram CANNOT be obtained from the rectangle P by means of:

A *a reflection about an axis in the plane of the page*

B *a rotation in the plane of the page*

C *a translation*

D *a translation followed by a reflection*

Circle the correct answer.

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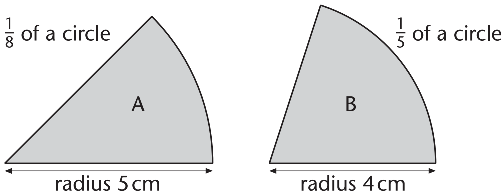
7.2 Interpret maps and scale drawings.

7.3 Visualise the effect of transformations on a plane figure; know that the image of a planar figure under rotation or reflection is congruent to the original figure before rotation or reflection and that every circle is similar to any other circle. (See also GM 7.1.)

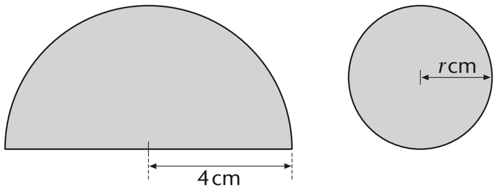
The diagram shows parts of two circles, sector A and sector B.



a. *Which sector has the bigger area?*

b. *The perimeter of a sector is made from two straight lines and an arc. Which sector has the bigger perimeter?*

A semicircle, of radius 4 cm, has the same area as a complete circle of radius r cm. What is the radius of the complete circle?



Not drawn to scale

7.4 Draw the plan and elevation of two-dimensional projections of three-dimensional rectilinear objects. (See also GM 6.1.)

8 Use vectors

8.1 Consider coordinate systems as grids for moving around space in two or three dimensions; understand position vector, unit vector and components of a vector.

A particle is at the point (6, 2)*. What is its position vector in terms of the unit vectors **i** and **j** in the x- and y-directions respectively? Calculate the length (magnitude) of this vector.*

Vectors

In three dimensions, vectors provide the natural language to place and displace figures in space. They also link with Cartesian coordinate systems.

Unit vectors

Unit vectors in three mutually perpendicular directions are usually written as ***i***, ***j*** and ***k***.

8.2 Interpret a translation as a vector displacement; know that a vector displacement from A to B depends only on the starting point A and the finish point B and not on intermediate steps from A to C to D to … to B, and that the vector sum of all these separate displacements from A to B is equivalent to the resultant displacement from A to B directly.

8.3 Add and subtract two vectors in up to three dimensions and draw corresponding vector diagrams.

Four vectors $\mathbf{a}$, $\mathbf{b}$, $\mathbf{c}$ *and* $\mathbf{d}$ *are given by* $\mathbf{a} = 2\mathbf{i} - 3\mathbf{j}$, $\mathbf{b} = 5\mathbf{j} + \mathbf{k}$, $\mathbf{c} = 4\mathbf{i} - 7\mathbf{k}$ *and* $\mathbf{d} = 3\mathbf{i} + \mathbf{j}$. *Find* $\mathbf{a} + \mathbf{b}$, $\mathbf{b} - \mathbf{c}$, $\mathbf{a} - \mathbf{b} - \mathbf{c}$. *Draw vector diagrams to represent* $\mathbf{a} + \mathbf{d}$ *and* $\mathbf{a} - \mathbf{d}$. *What are the components of these two vectors in the **i** and **j** directions?*

8.4 Multiply a vector by a scalar and know that this amounts to stretching the vector; calculate the magnitude and direction of a vector; use vectors to calculate displacement and velocity in a range of contexts.

A particle moves with constant velocity from A to B. Its position vector at A is $\mathbf{a} = \mathbf{i} + \mathbf{j}$ *and its position vector at B is* $\mathbf{b} = 5\mathbf{i} - 7\mathbf{j}$. *Calculate the vector displacement from A to B. If distance is measured in metres, show that the distance from A to B is* $4\sqrt{5}$ *metres. The particle takes 2 seconds to move from A to B. What is its velocity?*

8.5 Use the scalar product of two vectors to calculate the angle between the vectors and the scalar product of a vector with itself to find the magnitude of the vector.

*Find the magnitude of each of the vectors **a** and **d** in the example in GM 8.3 above. Calculate the angles between these vectors.*

8.6 Solve physical problems using vectors.

$\mathbf{i}$ and $\mathbf{j}$ are unit vectors in the east and north direction respectively. A ship has position $\mathbf{r} = 3\mathbf{i} + 4\mathbf{j}$ at 1200 hours. It then moves with constant velocity $\mathbf{v} = 4\mathbf{i} - 5\mathbf{j}$. The velocity is measured in kilometres per hour. What is the speed of the ship (the magnitude of its velocity)? What is the position of the ship at 1500 hours?

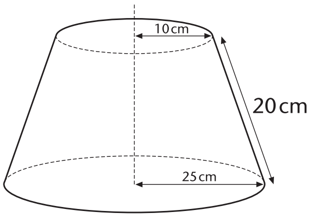
A particle of mass m kilograms is moving with constant acceleration $\mathbf{a}$, measured in metres per second per second. The total external force $\mathbf{F}$ acting on the particle is measured in newtons, and is the vector sum of the individual forces acting on the particle. The relationship between $\mathbf{F}$ and $\mathbf{a}$ is given by Newton's second law of motion and is $\mathbf{F} = m\mathbf{a}$.

A particle of mass 2 kg is acted upon by two forces $\mathbf{F}_1 = \mathbf{i} - \mathbf{j}$ and $\mathbf{F}_2 = 3\mathbf{j}$. Find the acceleration of the particle and give its magnitude.

9 Use a range of measures and compound measures to solve problems

9.1 Calculate lengths, areas and volumes of geometrical shapes.

A light shade is in the form of a frustum of a right cone. The radius at the top of the shade is 10 cm and the radius at the bottom is 25 cm. Find the surface area of the material used for the light shade.



9.2 Use approximation methods to calculate the area of an irregular two-dimensional flat surface and the volume of a prism with a constant, but irregular-shaped, cross-section.

9.3 Solve problems involving compound measures, using appropriate SI units and dimensions.

In one and a half hours a car uses 8 litres of petrol when travelling at a speed of 70 kilometres per hour. What is the petrol consumption in litres per kilometre?

Compound measures

Include measures of power, average speed and acceleration, measures of rate (such as rate of growth of income), and population density. Link where relevant to work in science, technology and the social sciences.

Probability and statistics

By the end of Grade 12, students arrive at conclusions from the formulation of a problem to the collection and analysis of data in a range of situations. They use secondary data from published sources, including the Internet. They use ICT to calculate statistical quantities and to produce a range of graphs, charts and tables to present and justify their findings. They calculate measures of spread, including the variance and standard deviation. They construct histograms and plot cumulative frequency distributions, using grouped continuous data if necessary. They understand that a random variable has a range of values that cannot be predicted with certainty and investigate common examples. They measure the empirical probability (relative frequency) of obtaining a particular value of a random variable. They use a simple mathematical model to calculate the theoretical probability of obtaining a particular outcome for a random variable associated with a set of events. They calculate probabilities of single and combined events, and understand risk as the probability of the occurrence of an adverse event. They use tree diagrams to represent and calculate the probabilities of compound events when the events are independent and when one event is conditional on another. They use simple simulations and consider trends over time using a moving average.

Probability and statistics

Students should know that statistics is the branch of mathematics used to predict the outcomes of large numbers of events when these outcomes are uncertain, and that probability lies at the heart of statistics. They should be aware of the uses of statistics in society and recognise when statistics are used sensibly and when they are misused or likely to be misunderstood.

Students should:

10 Collect, process, represent, analyse and interpret data and reach conclusions

Sampling

10.1 Know that:

- it is important to choose representative samples;
- in a random sample there are chance variations;
- in a biased sample there are systematic differences between the sample and the population from which it is drawn;

and locate obvious sources of bias within a sample.

Introductory statistical techniques

10.2 Plan surveys and design questionnaires to collect meaningful primary data from samples (including data collected in other subjects, such as science, geography or history) in order to make estimates of, or test hypotheses about, quantities or attributes characteristic of the population as a whole.

10.3 Formulate problems using secondary data from published sources, including the Internet.

10.4 Calculate measures of central tendency such as the arithmetic mean and the median.

10.5 Calculate measures of spread, including the variance and standard deviation.

Find the mean and median salaries of the group of workers in Qatar whose weekly salaries in riyals are given in the table below.

Salary (QR)	250	300	350	400	450	500	550	600
Frequency	5	11	20	31	18	12	7	3

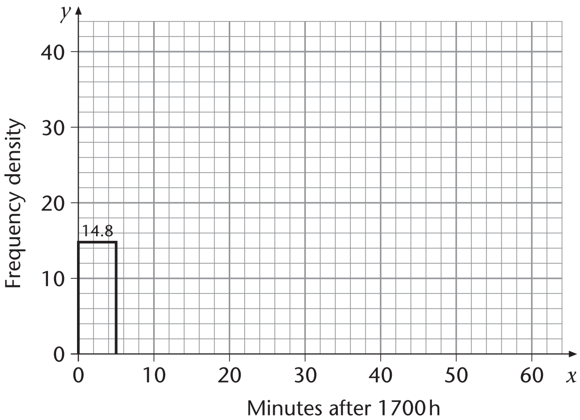
Which average is the most representative for these workers? Justify your answer. Use statistical functions on a calculator to calculate the standard deviation for the salaries in this group. What information does this convey?

10.6 Construct (relative frequency) histograms and know that the area of each block of the histogram represents the frequency of occurrence of the respective class interval associated with the block; plot cumulative frequency distributions, using grouped continuous data if necessary.

The table below shows the number of cars leaving a car park during the periods given.

Number of minutes after 1700 h	$0 \le n < 5$	$5 \le n < 10$	$10 \le n < 20$	$20 \le n < 50$	$50 \le n < 60$
Number of cars leaving	74	115	248	1174	189

Complete the histogram to show the information in the table.
Write the frequency density above each rectangle of the histogram.



The value 14.8 on the histogram is the frequency density for the period $0 \le n < 5$ minutes. Explain what is meant by frequency density with regard to cars leaving the car park.

10.7 Make inferences and arrive at conclusions from the formulation of a problem to the collection and analysis of data in a range of situations; use a range of statistics and graphs, charts and tables to present and justify findings.

Histograms and cumulative frequency distributions

Include the terms *frequency*, *frequency distribution*, *frequency density*, *relative frequency* and *relative frequency distribution*.

Include also the terms *range, percentile, interquartile range, semi-interquartile range*, and *mode, modal class, modal frequency*.

11 Understand random variables and calculate probability

11.1 Know that all probability values lie between 0 and 1, and that the extreme values correspond respectively to impossibility and certainty of occurrence.

11.2 Understand that a random variable has a range of values that cannot be predicted with certainty, and investigate common examples of random variables; measure the empirical probability (relative frequency) of obtaining a particular value of a random variable.

11.3 Use a simple mathematical model to calculate, for a particular set of events, the theoretical probability of obtaining a particular outcome for a random variable associated with those events.

Two six-sided dice, each numbered from 1 to 6, are thrown and the total score on the two dice is found. Assuming that either dice is equally likely to show any of its six faces, what is the probability that the total score is greater than 4 and less than 10?

Distributions

Include as models the rectangular distribution and the binomial distribution, and simple applications of these.

Assume that it is equally likely for a woman to give birth to a girl as it is to give birth to a boy. What is the probability that a woman with six children has four girls and two boys? What is the probability that if another woman has four children they are all boys? A woman with three daughters is going to have a fourth child. What is the probability that the fourth child will be boy?

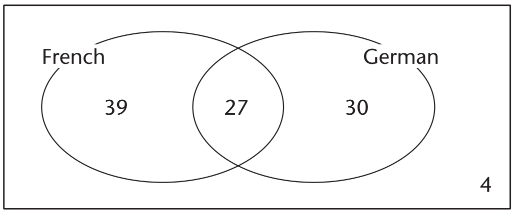
A company makes computer disks. It tested a random sample of disks from a large batch. The company calculated the probability of any disk being defective as 0.025. Naima buys two disks.

a. *Calculate the probability that both disks are defective.*

b. *Calculate the probability that only one of the disks is defective.*

c. *The company found three defective disks in the sample they tested. How many disks were likely to have been tested?*

100 students were asked whether they studied French or German. 27 students studied both French and German.



a. *What is the probability that a student chosen at random will study only one of the languages?*

b. *What is the probability that a student who is studying German is also studying French?*

A piece of string is 12 centimetres long. It is randomly cut into two pieces. What is the probability that one piece has length greater than 9 cm?

11.4 Understand *risk* as the probability of occurrence of an adverse event; investigate some instances of risk in everyday situations, including in insurance and in medical and genetic matters.

The probability of dying of cancer is $^1/_3$. What is the probability that if three people are chosen at random two of them will die of cancer? What is the probability that none of them will die of cancer?

Probability of combined events

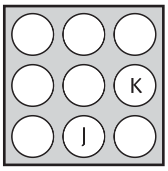
11.5 Understand when two events are *mutually exclusive*, and when a set of events is *exhaustive*; know that the sum of probabilities for all outcomes of a set of mutually exclusive and exhaustive events is 1, and use this in probability calculations.

11.6 Know that when two events A and B are mutually exclusive the probability of A or B, denoted by $P(A \cup B)$, is $P(A) + P(B)$, where $P(A)$ is the probability of event A alone and $P(B)$ is the probability of event B alone.

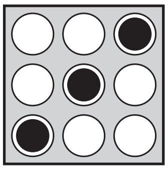
Mona has a chance of 1 in 4 of passing on a particular genetic condition to one of her children. Mona has three children. Calculate the probability that two of the children will inherit the condition. What is the probability that none of her children will inherit the condition? What is the probability that at least one her children will inherit the condition?

A computer game has nine circles arranged in a square. The computer chooses circles at random and shades them black.

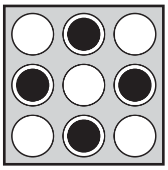
a. *At the start of the game, two circles are to be shaded black. Show that the probability that both circles J and K will be shaded black is* $^{1}/_{36}$.



b. *Halfway through the game, three circles are to be shaded black. Here is one example of the three circles shaded black in a straight line. Show that the probability that the three circles shaded black will be in a straight line is* $^{8}/_{84}$.



c. *At the end of the game, four circles are to be shaded black. Here is one example of the four circles shaded black forming a square. What is the probability that the four circles shaded black form a square?*



11.7 Know that two events A and B are *independent* if the probability of A and B occurring together, denoted by $P(A \cap B)$, is the product $P(A) \times P(B)$.

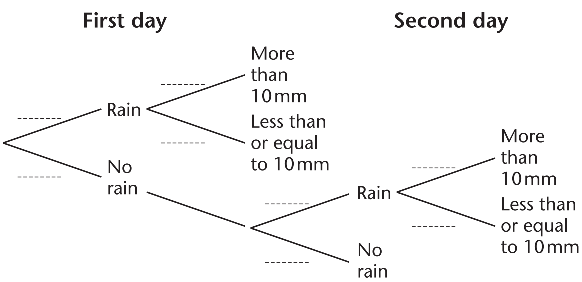
In a class of 35 students, the probability that a student picked at random is taller than 1.8 metres is 0.2 and the probability that the student wears spectacles is 0.3. What is the probability when three students are chosen at random that two are over 1.8 metres in height and that one of them wears spectacles?

Show that two events A *and* B *are mutually exclusive when* $P(A \cap B) = 0$.

11.8 Use tree diagrams to represent and calculate the probabilities of compound events when the events are independent and when one event is conditional on another.

On a tropical island the probability of it raining on the first day of the rainy season is $^{2}/_{3}$.
If it does not rain on the first day, the probability of it raining on the second day is $^{7}/_{10}$.
If it rains on the first day, the probability of it raining more than 10 mm on the first day is $^{1}/_{5}$.
If it rains on the second day but not on the first day, the probability of it raining more than 10 mm is $^{1}/_{4}$.

You may find it helpful to fill in this tree diagram before answering the questions below.



a. *What is the probability that it rains more than 10 mm on the second day, and does not rain on the first?*

b. *What is the probability that it has rained by the end of the second day of the rainy season?*

c. *Why is it not possible to work out the probability of rain on both days from the information given?*

20 per cent of the population of a country has a particular disease. A test can be given to members of the population to help determine whether or not they have the disease. The probability that the test gives a positive identification for those that have the disease is 0.7. But there is a 0.1 chance that a patient who does not have the disease still registers positive on the test. Find the probability that an individual selected at random tests positive, but does not have the disease.

Another person is chosen at random. Calculate the probability that the test result for this person is positive.

12 Calculate moving averages

12.1 Consider trends over time and calculate a moving average.

Find out about the cost, in Qatar, of a barrel of crude oil over the period January 2000 to March 2004. Analyse the data over periods of three months and compare the moving average price per barrel. Discuss your findings.

13 Simulation

13.1 Use coins, dice or random numbers to generate models of random data.

Do an investigation using random numbers to simulate waiting times at a doctor's surgery.

Random numbers

These can be generated on scientific or a graphics calculator using the RND and RAN function keys.

14 Use of ICT

14.1 Use a calculator with statistical functions to aid the analysis of large data sets, and ICT packages to present statistical tables and graphs.

ICT opportunity

A range of ICT applications can support data handling.

